ARE YOU ABOVE AVERAGE?

When we say someone is above average, what do people think? I put this question to a group of people recently and found many of them describing above average as being better than 50% of the people. It took quite a while to convince this group that you could be above average and in the bottom half of a class.

Even when presented with the following example, people doubted their eyes and kept looking for a trick.

Example:
Suppose you had nine students in a class and they scored marks of 10, 10, 20, 60, 70, 70, 70, 70 and 70. The class average would be 50. A student who scored 60 would be above average and yet in the bottom half of the class.

Perhaps we need to ask more of our students if you can be above average and in the bottom half of the class or even the reverse.

Knowledge of the median, rather than the mean, can be used to determine if you are in the top half or the bottom half of the distribution. Could we use simplified box-and-whisker plots to summarise assessment information? It would certainly require a significant community education program to enable these graphs to be interpreted.

The above data would be represented on a simplified box-and-whisker plot as follows.

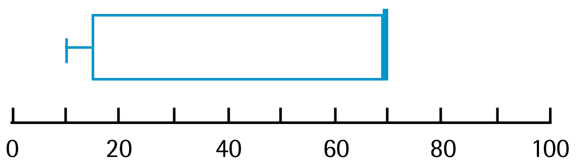


Fig. 1. Simplified box-and-whisker plot

It is not difficult to see that the spread of scores is quite wide and that the maximum value also corresponds to the median score of 70. The distribution of the scores is "heavy" on the top end. An individual student's performance could be located on the box-plot by a marker, such as a star or a circle, to suggest some sense of associated error.

The means of comparison provided through the use of a simplified box-and-whisker plot could be either an individual compared with the cohort or an individual compared with an expected standard. The latter will need to wait until we have clear ways of recording performance against the expected standard of the syllabus.

Skew

We rarely use different graphs to show the same information. This is usually because we make a decision as to which graph is the best way to present the information. As different graphs have different strengths it can sometimes be useful to present information using a pair of graphs. In the above simplified box-and-whisker diagram, the location of the median at the top of the box suggests that the data are skewed. The histogram is most often used to display the "shape" of the distribution. A histogram for the above data is shown in Figure 2.

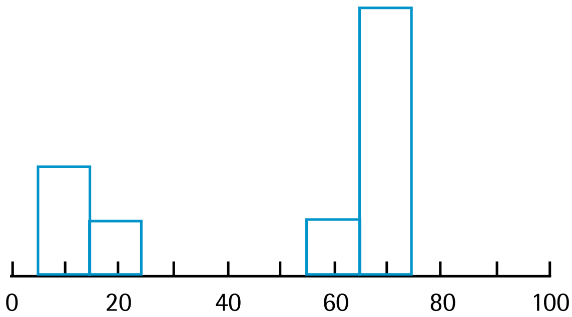


Fig. 2. Histogram of scores

A distribution is said to be skewed to the right (or positively skewed) if most of the data are clustered together at the lower values and tail off slowly towards higher values, as in Figure 3.

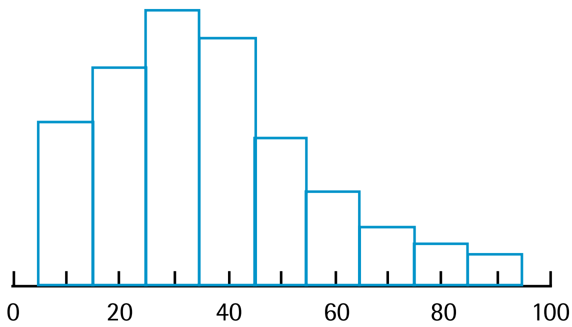


Fig. 3. Skewed to the right

Students often reverse the descriptions for skewed distributions. The description of skewed to the left or right refers to the tail of the distribution, not to the location of the "bump". Figure 4 shows a distribution that is skewed to the left.

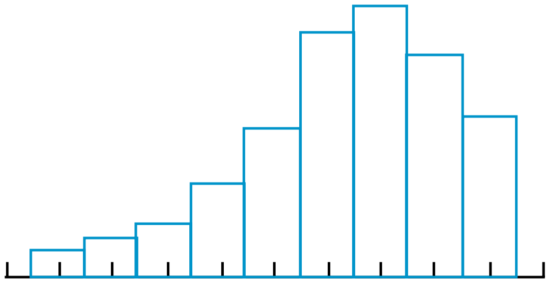


Fig. 4. Skewed to the left

Distributions that are skewed to the right or left are quite common when the data have a natural bound. For example, waistline measurements for a group of people at a fitness class or reaction times tend to have natural bounds.

A simplified box-and-whisker plot will also give an indication of whether the distribution is skewed. If the median is in the middle of the box and the lines to the extremes (sometimes called whiskers) are about the same length, then the data are symmetrical. If the median is near one end of the box or if the whiskers are of different lengths, then the data are skewed.

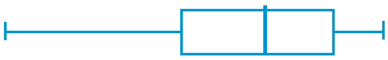


Fig. 5. Box-and-whisker plot skewed to the left

In the General Mathematics course, students encounter skewed distributions in the HSC topic DA5: *Interpreting sets of data*. It would be a good idea to build a basis for the idea of a normal distribution (DA6) and skewed distributions through the Preliminary course units DA3: *Displaying single data sets* and DA4: *Summary statistics*. Asking questions relating to the location of the average in a distribution and linking this to graphical displays will build the basis for understanding the effect of the shape of a distribution. Questions can be quite brief, such as "Can you be above average yet in the bottom half of the class?"

The meaning of summary statistics is often clearer when they are linked to graphical displays of data. For skewed data, the usual measures of location (mean, mode, median) will give different values. When the mode < median < mean, the distribution will be skewed to the right.

As well as investigating the shape and level of skew of distributions, DA5: *Interpreting sets of data* also explores outliers in small data sets and their effects on the mean, mode and median. An *outlier* is commonly taken as any score that is more than one and a half times the interquartile range above the upper quartile (Q_3) or below the lower quartile (Q_1).

USING STATISTICS FUNCTIONS ON EXCEL

With students and teachers in all state high schools having access to the full suite of software available in Microsoft Office, it is a good time to become familiar with some of the functions that can be used on Excel. Excel is a very powerful spreadsheet program that also has a range of statistical functions available. Yet Excel is not designed as statistical software, and some of the functions and charts contained within the program have unusual definitions.

The QUARTILE function

The QUARTILE function is designed to return any of the quartiles of a set of scores. We are often interested in finding Q_1 (the first quartile or 25th percentile), Q_2 (the second quartile, 50th percentile or median) or Q_3 (the third quartile or 75th percentile). The QUARTILE function in Excel also returns the value of Q_0 (the 0 percentile or minimum score) and Q_4 (the 100th percentile or maximum score). It should be an ideal function to produce a five-figure summary of data.

The QUARTILE function can be used with direct entry of the data or you can specify a range of cells in the spreadsheet that the function will use as its argument (subject matter). If you wanted to use this function to find Q_1 for the data 1, 2, 4, 7, 8, 9, 10, 12 it would appear as in Figure 1.

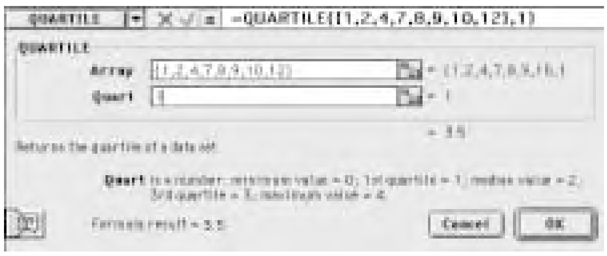


Fig. 1. Lower quartile

The result of this calculation is rather surprising. To gain an understanding of how Excel attempts to calculate a quartile we can change the third data point from 4 to 6, as in Figure 2.

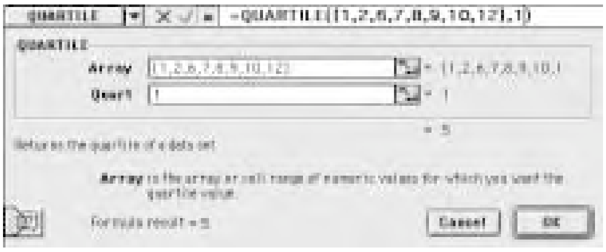


Fig. 2. Lower quartile recalculated

The program is clearly not determining the average of the second and third scores. If we use four data points 1, 2, 3, 4, Excel calculates the 25th, 50th and 75th percentiles as 1.75, 2.5 and 3.25 respectively. Excel's method of calculating the median is consistent with the way we would teach it. However, Excel's method of determining quartiles certainly isn't. For the data set 0, 0, 0, 1000, the first and second quartiles are calculated as 0 but the QUARTILE function provides a value of 250 for the third quartile.

Unfortunately the QUARTILE function draws upon subroutines from the PERCENTILE function. Instead of locating scores that divide the distribution into quarters, the QUARTILE function attempts to locate scores that divide the distribution into hundredths and then use 25 hundredths and 75 hundredths as Q_1 and Q_3. If you have a large number of scores, the QUARTILE function will behave quite normally. With small numbers of scores the PERCENTILE function interpolates values. This can produce some unusual values when small numbers of scores are involved.

Calculating quartiles without using interpolation

We can use other formulae in Excel to determine quartiles without relying on interpolating percentiles. The quartiles, including the median, divide the scores roughly into quarters. Consequently, we need to consider the remainders produced by dividing the number of scores by four. Diagrammatically, the cases we need to consider can be represented as follows.

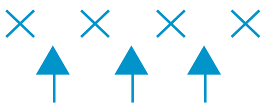


Fig. 3. 4k

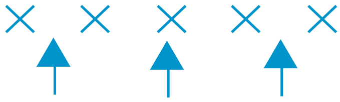


Fig. 4. 4k + 1

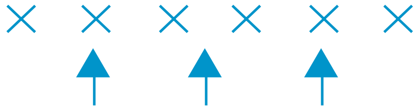


Fig. 5. 4k + 2

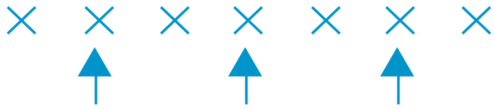
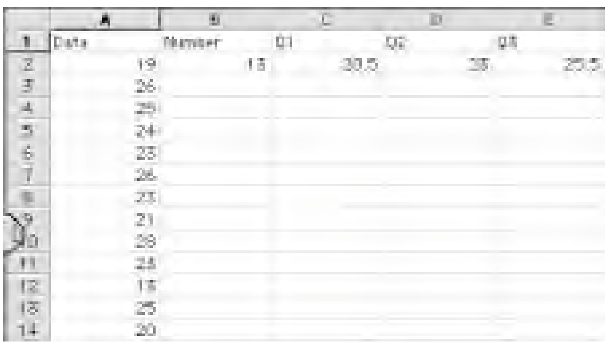


Fig. 6. 4K + 3

The lower quartile is found by determining the middle score of all the scores **below** the median. Similarly, the upper quartile is found by identifying the middle score of all the scores **above** the median. Determining the upper and lower quartiles thus reduces to two cases. If we divide the number of scores by 4 and the remainder is a half or three-quarters, then the quartiles will be scores. The upper and lower quartiles will then be the scores that are the integer part of ($\frac{n+2}{4}$) from each end. If the remainder when the number of scores is divided by 4 is zero or a quarter, then the upper and lower quartiles are each the average of two scores. The position of the scores is given by the integer part of $\frac{n}{4}$ and the next score from each end.

Using these ideas we can create an Excel spreadsheet that will generate the upper and lower quartiles without using percentiles. The spreadsheet will be set out to allow up to 100 data values to be entered.



	A	B	C	D	E
1	Data	Number	Q1	Q2	Q3
2	19	13	20.5	25	25.5
3	26				
4	25				
5	24				
6	23				
7	26				
8	23				
9	21				
10	28				
11	24				
12	18				
13	25				
14	20				

Fig. 7. Spreadsheet generating quartiles

The first column has been used to enter the data. The second column counts the number of data points entered. Cell B2 contains the formula COUNT(A2:A101) that counts how many of the 101 cells in the first column have numbers entered. All of the entries in the first row are text entries.

Cell D2 is used to calculate the median using the formula MEDIAN(A2:A101). This formula calculates the median of all numbers in the array of cells from A2 to A101.

Cell C2 calculates the lower quartile. The data do not need to be entered in order. The formula entered in C2 uses a logical function IF, the integer function INT, and a function to find a specified kth smallest value SMALL. The IF function evaluates an expression and returns one value if true and another if false. This function allows the lower quartile to be evaluated using two different methods, depending upon the number of scores.

The formula entered in cell C2 is as follows:

= IF(((B2/4)-INT(B2/4)) < 0.5,

(SMALL(A2:A101,INT(B2/4)) + SMALL(A2:A101,INT(B2/4) + 1))/2,

SMALL(A2:A101,INT(B2/4) + 1)).

This rather complex looking formula has three parts within the IF function. The first part determines if the number of scores is a multiple of four or one more than a multiple of four. If this condition holds, the cell returns the average of two of the data values. Should this condition not be met, the cell returns a particular value, in this case, the 4th smallest value.

Cell E2 contains essentially the same formula as cell C2 with the SMALL function replaced by the LARGE function.

Having set up the above spreadsheet, it is quite a simple process to add in another cell to calculate the mean of the scores (AVERAGE(A2:A101)). Using this spreadsheet, students can explore questions about the distribution. For example, the median always falls between the two quartiles but does the mean always fall between the upper and lower quartiles?

Many graphics calculators also have built-in statistics functions to make calculating the upper and lower quartiles quite easy.

DEVELOPING ASSESSMENT TASKS

The previous HSC supplement in **CURRICULUM SUPPORT**, Vol. 5, No. 1, p. 9 outlined how to design an assessment program and map subject outcomes.

Points to consider in the development of the assessment program include:

- Components and weightings are included.
- All course outcomes are included.
- There is a range of task types consistent with the outline in the syllabus.
- Each assessment task is appropriate to the outcomes being assessed.
- The total assessment program allows for the demonstration of the range of achievement of the outcomes.
- There are about 3-5 tasks.
- Individual tasks are worth between 10% and 40% of the total assessment mark.
- Later tasks carry more weight, so that the final assessment mark is more likely to reflect the standard achieved.
- Measures of objectives and outcomes from the affective domain (values and attitudes) are not part of the assessment marks submitted to the Board of Studies.

Having mapped the course outcomes, identified the range of appropriate tasks to be used, taken into account the components and weightings, and decided on the most appropriate task type to be used, teachers need to develop the task.

In developing a quality assessment task it is important to consider that all tasks:

- are integral to the overall teaching and learning program
- have a direct link with syllabus outcomes
- are explicit in what students are required to do
- are valid and reliable
- are fair and equitable
- allow each student to demonstrate his or her level of achievement
- are time efficient and manageable
- have clear and explicit criteria for making judgements

- have identified marking guidelines
- provide opportunities for meaningful feedback.

The General Mathematics syllabus clearly outlines the main outcomes related to each unit of work. Designing assessment tasks related to specific units of work will readily identify which outcomes are being addressed. The marking criteria will usually be based upon the identified skills, knowledge and understanding that students are expected to acquire. These are described within the syllabus.

The following assessment task was originally designed as a teaching task. It addresses the difference between the arithmetical mean and the median. Effective and informative assessment should link to well-structured teaching and learning activities. The task will be valid if it reflects the actual intent of teaching and learning activities developed from the syllabus.

The marking guidelines provide guidance on the assignment of marks to student responses in relation to each of the criteria identified.

ASSESSMENT TASK: Are you above average?

Syllabus links

DA3: Displaying single data sets

DA4: Summary statistics

Outcomes addressed

A student:

- represents information in symbolic, graphical and tabular forms (P4)
- determines an appropriate form of organisation and representation of collected data (P9)
- justifies his or her response to a given problem using appropriate mathematical terminology (P11)

The task

There are 18 students in a class. All of the students complete a test that is marked out of 100. Is it possible for one of these students to achieve above the class average on the test and yet finish in the bottom half of the class?

Explain your answer using an example that includes a frequency table and at least one carefully selected graph. Indicate why you chose this type of graph.

Is it possible for the reverse situation to occur, that is, to be below average and yet finish in the top half of the class? Explain why not or describe how an example could be created.

Marking guidelines

Your task will be assessed on how well you:

- explain the relationship between a summary statistic and a distribution (2 marks)
- calculate the mean of data sets (1 mark)
- link types of data with appropriate displays (1 mark)
- describe the strengths or weaknesses of various graph types. (1 mark)

This task is quite open in that it has a range of possible correct responses. To ensure that the full range of students are able to carry out the task, it should build on students' within-class experiences of constructing data sets with given parameters. For example, tasks such as "Find five scores that have an average of 10," or "Find five scores with a median of 12" would be useful to develop the necessary skills.

A range of responses to the task is possible. One response could be as follows.

Scores	Frequency	fx
10	2	20
20	1	20
30	1	30
40	1	40
50	1	50
60	1	60
70	1	70
80	4	320
90	4	360
100	2	200
	$\sum f = 18$	$\sum fx = 1170$

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{1170}{18} = 65$$

The student who scores 70 is above the class average of 65 but finishes 11th out of 18 and is in the bottom half of the class.

Appropriate ways of displaying this data could include a dot plot (fig. 1), histogram or even a box-and-whisker plot (fig. 2).

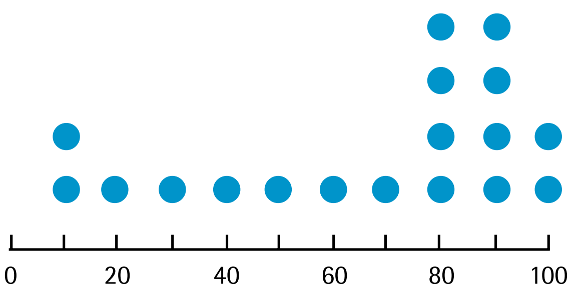


Fig 1. Dot plot

Sometimes the dots in a dot plot are replaced with "x". A dot plot is equivalent to a histogram, in that it shows the shape of the distribution.

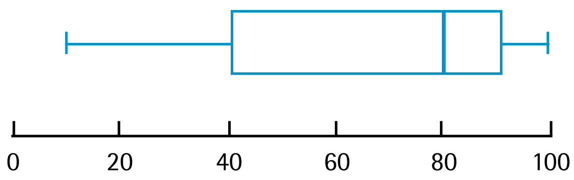


Fig 2. Box plot

In developing the task teachers should ask themselves:

- Does the task fit into the overall teaching and learning program?
- What outcomes do I want to assess?
- What type of task will best assess student achievement in this context?
- How will I mark the task to reflect student achievement of the outcomes assessed by the task?
- Does the wording of the task provide clear directions to students about what they are expected to do?
- Does the task allow students to show a range of achievement?

The task on p. 15 is designed to give students practice at using similarity associated with scale drawing to solve practical problems.

The marking criteria provide the link between the syllabus content and the outcomes. The use of explicit marking criteria is a significant change and needs to be modelled during the Preliminary course. Allocating marks to the criteria rather than the parts of the question is essential in making sure that the assessment is closely tied to the expected standard. The main caution with this process is to be certain that, in doing this, you do not fragment authentic assessment. The criteria should come from the task rather than designing individual components to match each criterion.

Assessment criteria can also be developed further to show the finer detail associated with the range of different performances. For example, "develop a scale drawing" could be exemplified by providing the characteristics of performance that would produce 1 through to 5 marks. This would improve the marking guidelines for this task by being more explicit as to what features of a scale drawing are valued.

ASSESSMENT TASK: Line of sight

Syllabus links

M3: Similarity of two-dimensional figures

Outcomes addressed

A student:

- applies mathematical knowledge and skills to solving problems within familiar contexts (P2)
- performs calculations in relation to two-dimensional and three-dimensional figures (P6)
- determines the degree of accuracy of measurements and calculations (P7)
- justifies his or her response to a given problem using appropriate mathematical terminology (P11)

The task

In designing some rooms, it is important to provide privacy to certain areas. The following en suite design uses a screen (AB) to provide privacy to the toilet section even if the door is open.

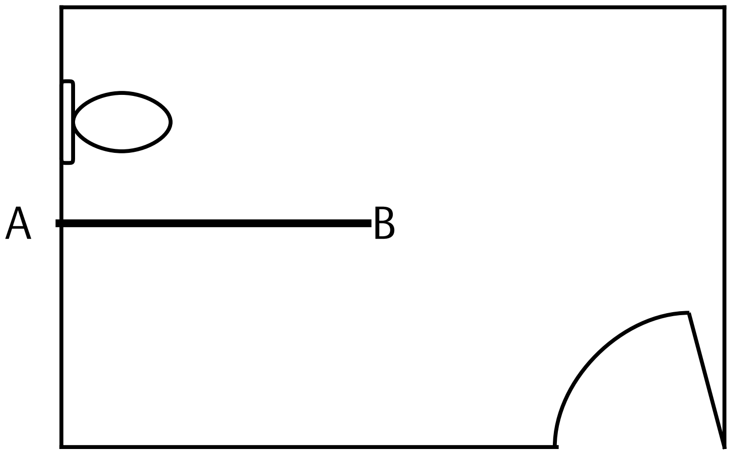


Fig 3. En suite design

The en suite is twice as long as it is wide. If the toilet extends 650 mm from the wall, create a scale drawing of the en suite. Use a standard door width and reasonable values for the length and breadth of the room. Mark the dimensions of the room and the scale you have used on your plan. Locate the screen AB half way along the wall. What is the minimum length of the screen AB that will provide privacy with the door open? Explain your answer, using diagrams and any necessary calculations.

Marking guidelines

Your task will be assessed on how well you:

- develop a scale drawing (5 marks)
- find scale factors of similar figures (2 marks)
- use relevant enlargement factors to calculate actual dimensions (2 marks)
- round calculated measurements to an appropriate number of significant figures (1 mark)