

# Count Me Into Space Angles

Teaching Angles by Abstraction:  
A professional development  
experiment in Year 3

2001 Report



# Count Me Into Space Angles

Teaching angles by abstraction:  
A professional development  
experiment in Year 3

2001 Report

A report prepared for the New South Wales  
Department of Education and Training

by

Associate Professor Michael Mitchelmore,  
Macquarie University

Dr Paul White, Australian Catholic University



The researchers are grateful for the insightful assistance of Anne Prescott in the collection, analysis and interpretation of the data on which this report is based.

Count Me Into Space Angles

Teaching angles by abstraction: A professional development experiment in Year 3  
2001 Report

© 2002 NSW Department of Education and Training  
Professional Support and Curriculum Directorate

**Restricted waiver of copyright**

The printed material in this publication is subject to a restricted waiver of copyright to allow the purchaser to make photocopies of the material contained in the publication for use within a school, subject to the conditions below.

1. All copies of the printed material shall be made without alteration or abridgement and must retain acknowledgment of the copyright.
2. The school or college shall not sell, hire or otherwise derive income from copies of the material, nor distribute copies of the material for any other purpose.
3. The restricted waiver of copyright is not transferable and may be withdrawn in the case of breach of any of these conditions.

ISBN: 0 7313 8154 8

SCIS: 1087549

*Count Me in Too* is an early number project initially designed by the New South Wales Department of Education and Training (DET) in 1996. It aims to assist teachers better understand how children learn mathematics by focusing on different strategies that children use when solving number tasks. The *Count Me Into Measurement* and *Count Me Into Space* projects in Years K–2 have since been developed along the same lines as the number project.

In 2001, the DET identified angles as a key area of student learning to be included in the *Count Me Into Space* project for Year 3. The angle concept is seen as an integral part of learning about space and a natural extension of the two key ideas included in the Years K–2 units on 2D and 3D space: Part-whole relationships, and orientation and motion. Angles are important parts of spatial objects which help to define their shape, and angles are also closely related to the idea of direction.

This report presents the details of the collaborative initiative between the Department of Education and Training and the researchers. The aims of the initiative were to evaluate the effectiveness of:

- professional development in the teaching of angles for a group of teachers
- a draft teaching and assessment package for angles in Year 3.

The unit was designed on the basis of theories which the researchers had developed from their previous research on the development of children's angle concepts.

## RATIONALE

### THE IMPORTANCE OF THE ANGLE CONCEPT

Along with length, angle is possibly the most important mathematical tool for describing and analysing physical space (for example, in construction, design and navigation). It also plays an important role in the development of geometry later in students' education. It is therefore vital that students obtain a good grounding in their concepts of angle at an early stage.

The angle concept is exceptional because it arises in so many different contexts. For example, angles are not only used to describe the shape of the corner of a geometrical figure but they are also used to specify a direction of movement, the direction of an object, an amount of turning or opening, and an inclination or slope. This makes angle an interesting topic to teach, because the teacher can draw on many examples in the students' environment. But, because angles arise in different ways in different situations, it also makes the angle concept quite difficult for students to learn.

### LEARNING ABOUT ANGLES

In Mitchelmore and White (2000), we showed that the easiest angles for children to learn about are what we call *2-line angles*, that is, angles in which both arms of the angle are visible. Examples are the angles formed by the corners of geometrical figures and by corners and intersections in our environment (for example, corners of a room, road intersections and the blades of a pair of scissors).

Another important group of angles is what we call *1-line angles*. In these angles, only one arm of the angle is clearly visible—the other line of the angle must be imagined or remembered. Objects which form 1-line angles include doors, windscreen washers and ramps. Our research indicated that these types of angle are much more difficult for children to identify than 2-line angles.

Many angles arise from situations in which neither arm of the angle is visible. Think, for example, of a rebounding or spinning ball, or how we describe directions. We call these *0-line angles*. Our research indicated that these types are very difficult for students to identify, even in Year 8.

In Mitchelmore and White (2000), we also showed that students' major difficulty in learning about angles is to recognise that the same idea is present in different situations. Even 2-line angles present some difficulties. Some 2-line angles are horizontal (e.g. the corner of a table top) and some are vertical (the corner of a window frame). Some are solid (furniture corners) and some are empty (lattice work). Some are fixed (picture frames) and some are movable (a pair of scissors). In some cases the two lines are easy to detect (table corners), in others they are more difficult to discern (objects with rounded edges). Children have to recognise that all these objects are similar in that they consist of two linear parts which cross or

meet at a point. They are also similar in that the relative inclination of these two parts has some significance: It defines the sharpness or *openness* of the object forming the angle.

Our research indicated that students can identify 2-line angles where the two lines are easy to detect as early as Year 2. Young students also seem to understand a number of 1-line angle situations (doors, clocks, ramps, etc.) very well—but they do not easily recognise that these situations can also be described using angles. To do so, the student must see the similarity between such 1-line situations and the 2-line situations in which they first met the idea of an angle. In order to recognise this similarity, they must be able to find suitable second lines in the 1-line situations so that they can be matched to the 2-line situations. However, 1-line situations vary greatly and the second line must be identified separately in each situation. To extend their ideas of angles from 2-line to 1-line situations, students must therefore develop their global understanding of angle into an analytical understanding.

The concept of angle is abstracted from the similarities between different angle situations. In fact, it is the *embodiment* of their similarity. The three critical features of this similarity are (1) two lines, (2) a point where the lines meet, and (3) a degree of openness between the lines. This is really a circular definition, because openness can only be defined in terms of angles! However, it is impossible to define the concept of angle otherwise. The best one can say is that it is that which is common to all those situations which we recognise as being similar in the way described above.

## TEACHING ABOUT ANGLES

In Mitchelmore and White (2000), we also proposed a method of teaching the angle concept based on our research. In this method, which we call *Teaching for Abstraction*, students are involved in activities which are intended to help them:

- understand various contexts thoroughly in terms of their angle features
- recognise the similarities between different angle contexts.

We have investigated the effectiveness of this method in one-to-one and small-group teaching situations (White & Mitchelmore, 2001). The current project represents the first test of Teaching for Abstraction in realistic classroom situations.

## PROCEDURE

### OVERVIEW

In collaboration with DET curriculum officers and consultants, the researchers designed an Angles Unit on the basis of their research described above. They then developed a corresponding teaching package (Mitchelmore & White, 2001) comprising (1) background information describing research on the angle concept and the idea of Teaching for Abstraction, (2) outlines of ten lessons, (3) some materials to resource the lessons, and (4) some assessment tasks. This package was then trialled in five schools by twelve teachers. These teachers also participated in two workshops, one before and one after the teaching of the unit.

### THE STRUCTURE OF THE ANGLES UNIT

The ten lessons covered the material in 2D Space Units 10 and 14 of the current *K–6 Mathematics Syllabus*. The focus was on Stage 2 Outcome S2.2(b) in the *Mathematics K–6 Outcomes and Indicators*, together with Outcomes WM2.1, WM2.3, VA2, VA6, and VA10. The aims of the lessons were for students to learn to recognise angles in corner, pivot, turn, and slope contexts. Students were expected to be able to:

- indicate the angle in typical situations using a bent straw or by drawing a diagram,
- match angles within and across contexts
- describe the size of angles in given situations.

A summary of the lessons is given in Table 1.

LESSON	DESCRIPTION	LESSON	DESCRIPTION
1.	<b>Pattern block corners</b> Matching and comparing block corners, introducing the word “angle”	6.	<b>Clocks</b> How a clock uses angles to show the time
2.	<b>More corners</b> Corners in the classroom, right angles	7.	<b>Doors</b> Interpreting the opening of a door in terms of angles
3.	<b>Comparing corners</b> Making an angle tester to informally measure corners	8.	<b>Slopes</b> Interpreting slope in terms of angles
4.	<b>Scissors</b> Using angles to describe the amount of opening of a pair of scissors	9.	<b>Angles, angles everywhere</b> Matching angles across all the situations investigated in this unit
5.	<b>Scissors-like objects</b> Measuring and matching angles in other objects with two parts connected by a pivot	10.	<b>Creating angles</b> An open-ended task involving a variety of different angles

Table 1. Angle lessons

The first five lessons of the unit dealt with 2-line angles. The emphasis was on matching angles from different situations (both fixed and movable) by placing one 2-line angle on another (called *superimposition*). Fixed 2-line angles were also combined into a primitive angle-measuring device. The culmination of this section was a discussion of the three critical features of an angle mentioned above.

The remaining five lessons dealt with 1-line angles and how to match them to the 2-line angles met in the first section. The idea was to interpret each position in terms of the angle formed between the object and some neutral position (12 o'clock on the clock, the closed position of the door, and the horizontal direction). The final two lessons challenged students to match all the 2- and 1-line angles they had met in the unit and to find further examples of angles from activities outside the mathematics KLA. No 0-line angles were included.

All of the lessons included small group activities. Most of these activities were accompanied by worksheets on which students recorded their findings. Further details of the lessons and the worksheets may be found in the *Lesson Evaluation* section below.

## ASSESSMENT TASKS

The assessment tasks were designed as individual interviews to be administered and scored by the teacher before and after teaching the unit. Changes in student responses could then be used as a measure of the effectiveness of the unit.

The assessment interview consisted of three tasks:

1. *Matching angles in pairs of objects.* Students were asked to open a pair of scissors to match a pattern block corner, to open a door to match a different pattern block corner and to turn a clock hand to match the slope of a ruler.
2. *Identifying lines and vertices of angles.* Students were asked where the angle was on a series of objects. Two of these were pattern block corners—the  $60^\circ$  angles on a trapezium block and an equilateral triangle. The other objects were a pair of scissors, a door, a clock and a sloping ruler.
3. *Finding angle sizes.* Students were asked to identify a right-angled block, move a clock hand through a right angle and then half a right angle and slope a ruler at half a right angle.

A recording sheet was provided on which teachers recorded students' responses before and after teaching the unit.

Before incorporating them into the teaching package, the assessment tasks were trialled by a DET curriculum officer and modified accordingly.

## **PARTICIPANTS**

Five DET schools agreed to participate in the trial of the Angles Unit. A total of 12 teachers was involved, 2 or 3 in each school. Most of the classes were Year 3, but there were a few Year 2/3 composite classes and one Year 3/4 class. In two classes, the teaching was shared between two teachers but, in both cases, all the mathematics teaching was done by the teacher who participated in the project.

District mathematics consultants assisted the teachers in each school.

## **TEACHER WORKSHOPS**

All the teachers who participated in the project attended two, one-day workshops, one before and one after teaching the unit. These workshops were organised by DET and led by the two researchers. The four district consultants also attended some of the sessions.

The first workshop took place on 22 June 2001. It consisted of three sessions in which the following activities took place:

- Teachers shared their experience in teaching angles.
- The researchers outlined recent research on students' understanding of angle concepts and presented the rationale and structure of the Angles Unit.
- The teachers worked through and commented on most of the proposed student activities in the unit.
- Teachers watched a video of a sample assessment interview and role-played the administration of the interview to each other.
- The researchers discussed the procedures for trialling the lesson materials, assessing students and reporting the results.

The second workshop took place on 21 September 2001, after all teachers had completed teaching the Angles Unit. The three sessions of this workshop covered the following topics:

- teachers' experience in teaching each lesson (suggestions for improvement were listed under the headings of management and learning issues)
- a preliminary analysis of the assessment results
- an overall evaluation of the Angles Unit and its rationale, and a discussion of how teachers had benefited from participating in the project
- collection of videos of student assessments, teachers' assessment reporting sheets and students' work samples.

## TEACHING THE UNIT

Each teacher was asked to identify a target group of eight students in her class, to administer the assessment interview to all eight students before and after teaching and to videotape the interviews. Teachers were also asked to compile a folder of work samples for each of these eight students.

It was intended that teachers should administer the pre-assessment in the week after the first workshop and start teaching the Angles Unit in the following week. However, as these were the last two weeks of Term 2, most teachers managed to complete the interviews in this time but did not start teaching the unit until Term 3.

All the teachers completed teaching all or most of the ten lessons in the time available, but to achieve this had occasionally to teach more than one lesson a week. One teacher finished by Week 7 of Term 3 before taking on maternity leave. In another school, students were also engaged in a research project on 2D space run by a different division of the DET. Only three teachers were able to teach all ten lessons.

All teachers completed the post-assessment interviews in the last week before the second workshop. They also all collected student work samples as requested.

## LESSON EVALUATIONS

The following observations derive from two sources: (1) comments made by the 12 teachers in the second workshop and on the unit evaluation form and (2) the work samples of 90 students.

Before looking at each lesson in detail, a general comment: When time ran out and a lesson had to be curtailed, it was the whole group discussion at the end of the lesson that was omitted. While recognising that this part of the lesson was important, the teachers in many instances had little choice but to stop at that point.

### LESSON 1: PATTERN BLOCK CORNERS

For this lesson, students were supplied with pattern blocks from a standard set consisting of a square, an equilateral triangle, a regular hexagon, a trapezium with angles of  $60^\circ$  and  $120^\circ$ , and two rhombuses (one with angles of  $60^\circ$  and  $120^\circ$  and one with angles of  $30^\circ$  and  $150^\circ$ ), each one of a distinctive colour. They were asked to make patterns out of blocks of the same colour by fitting the same angle around a central point. They then used these patterns to compare the size of the angles of the different pattern blocks.

The teachers felt this was a good lesson that had been enjoyed by the students. Some typical student responses are shown in Figure 1.

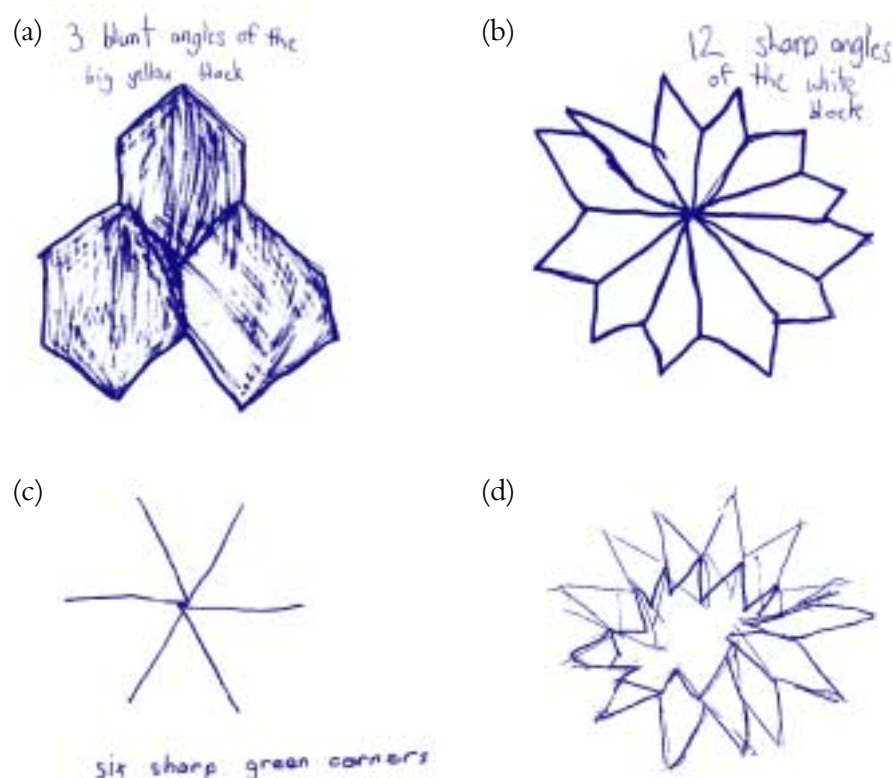


Figure 1: Examples of student work from Lesson 1

There was some confusion as to whether the purpose of the activity was to trace the whole block, as in Figures 1(a) and 1(b), or to concentrate on the lines from the central point, as in Figure 1 (c). Some students had considerable difficulty drawing the tile pattern, as illustrated in Figure 1(d).

Some students had difficulty understanding the task. Successful strategies which teachers used to assist students included:

- making the pattern on an overhead projector, with the pattern blocks pulled slightly apart
- calling the patterns *windmills* or *flowers*

Teachers also felt that more time for free exploration of the tile patterns would have been desirable, particularly given the fact that some blocks (the equilateral triangle and the  $30^{\circ}$ – $150^{\circ}$  rhombus) were so small they were difficult for the children to manipulate.

## LESSON 2: MORE CORNERS

The students had fun and enjoyed this lesson, in which they looked for angles in their classroom, copied the angles using a bent straw and drew sketches of the angles. “They had a wonderful time finding the angles.”

The problem of focusing on the whole shape rather than the angles was again evident (see Figure 2). When using the straw to show an angle, many drew the actual straw in 2 or 3 dimensions (usually by tracing it), even including the stripes.

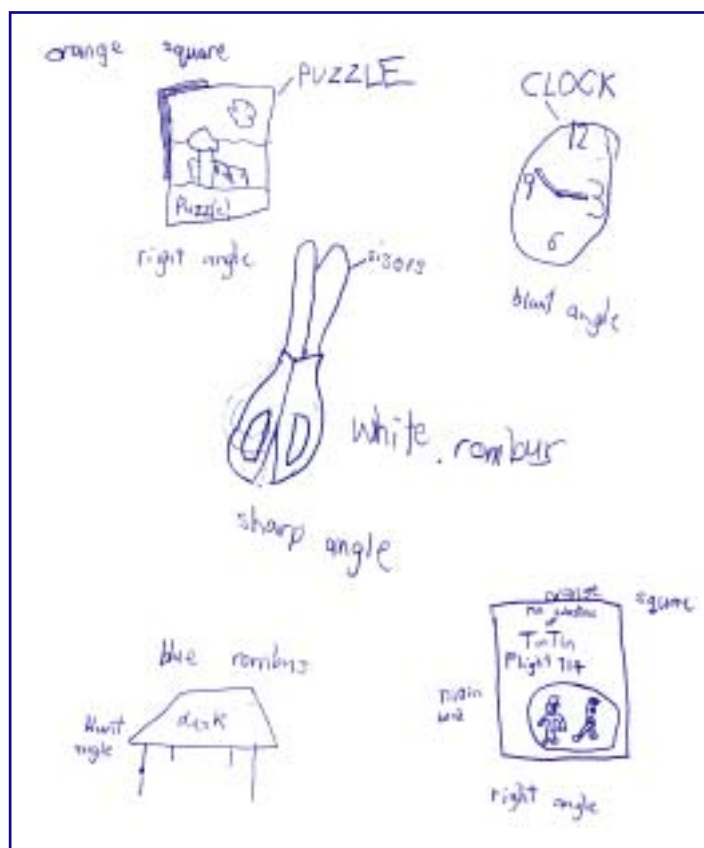


Figure 2: A student's sketch of angles in the classroom

Practical problems occurred because the straw bent in a curve and did not hold its shape once a student had formed the angle. Putting a pipe cleaner through the straw was seen as a solution to this problem. This addition gives a sharp point, maintains the flexibility of the straw, and keeps the angle reasonably fixed as the students move it to their page for drawing.

The teachers felt that the terms *acute* and *obtuse* should be introduced here, rather than the terms *sharp* and *blunt* used in the unit materials. One teacher, whose students were already using the terms *acute* and *obtuse*, was particularly disappointed. Most students already knew the term *right angle*.

### LESSON 3: COMPARING CORNERS

In this lesson, students made an angle tester by folding a sheet of paper into 12 equal angles at the centre, and used it to measure angles around the classroom. The students thoroughly enjoyed the lesson and most of the teachers' comments were generally favourable ("This is my absolute favourite.") However, one teacher found it difficult ("Get rid of this!").

The angle tester (Figure 3) was useful in demonstrating that the size of an angle is not dependent on the length of the lines. At the beginning of the lesson, the students were quite sure that the angles on the angle tester were all different. For example, numbering the lines by the hours on a clock, the angle between 11 and 12 seemed larger than the angle between 8 and 9. By the end of this lesson, most knew the angles were all the same.

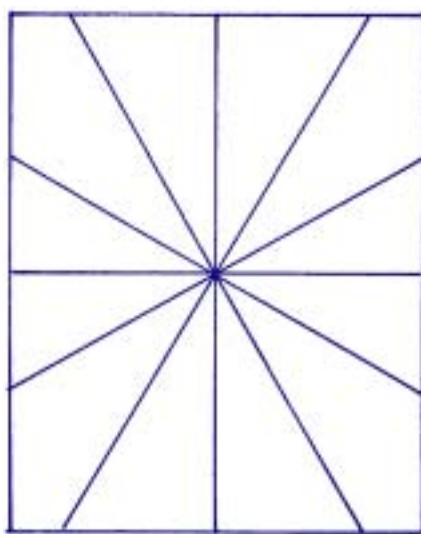


Figure 3: An angle tester made by folding a sheet of paper

There was considerable discussion about the angle testers: Making them was a time consuming exercise and the angle testers that were not folded accurately caused problems in subsequent lessons. However, when teachers took the students as a group through each fold and made sure all the students had a correctly folded their sheets of paper, there were no problems.

It appears from the work samples that most students were able to use the angle tester to measure the sizes of the pattern block corners. The main difficulty occurred in measuring the obtuse angles, particularly on the smallest block.

#### LESSON 4: SCISSORS

The students thoroughly enjoyed learning to interpret the opening of a pair of scissors in terms of angles and thought it neither too hard nor too easy. Students liked using the words *pivot* and *superimpose* and teachers liked the worksheet (see Figure 4).

A major difficulty arose in representing the opening of a pair of scissors by an angle. Theoretically the angle should have its vertex at the pivot, since this is the point about which the blades rotate. However, the unit materials did not make this clear. So, although most students identified the pivot easily, they often represented the angle of opening by the angle between the blades. For example, the angles drawn in Figure 4 are incorrect because their vertices are far away from the pivot.

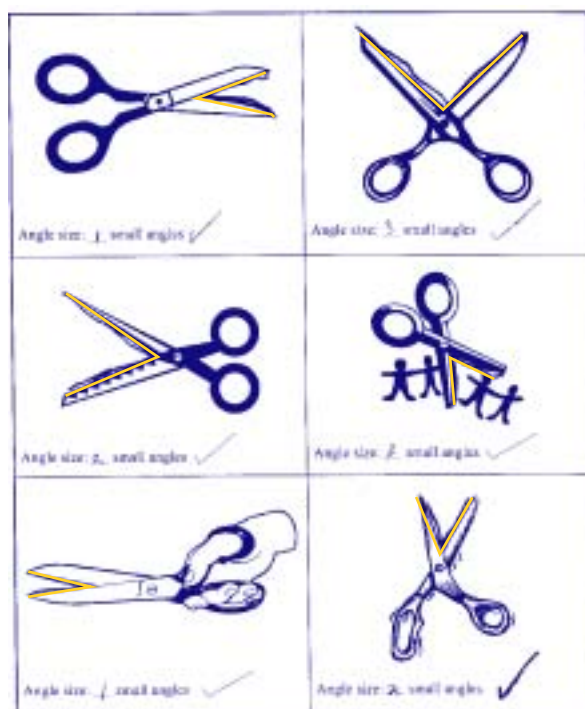


Figure 4: A completed worksheet for Lesson 4

In one activity in this lesson, students worked in pairs. One student opened a pair of scissors to the same angle as a pattern block corner, and their partner had to guess which block had been used. The students particularly enjoyed this game, and there was great delight when a student could inform their partner that they had chosen the correct angle but the wrong block. (For example, an angle of  $60^\circ$  could be made with a corner of either an equilateral triangle or one of the rhombuses.) It was during this activity that some students finally realised that the corners of different pattern blocks could be the same size.

## LESSON 5: SCISSORS-LIKE OBJECTS

In this lesson, students found further objects that open and shut like scissors and investigated the similarities between scissor-like objects and corners. Teachers showed a mixed reaction to this lesson; some found it quite easy to follow, whereas others left it out because they could not understand what was required.

Only 52 of the 90 students whose work samples were submitted completed the worksheet for this lesson. Those who did particularly enjoyed drawing a stick figure with a right angle at the elbow,  $30^\circ$  between arm and body, and  $60^\circ$  between the legs. (These angles were specified in terms of pattern block corners.) One student enjoyed demonstrating, using his own arms and legs, the stick figure he had drawn. Another student very proudly showed the class that his arm could straighten to “7 little angles” ( $210^\circ$ ). Figure 5 shows some student work. Most students drew at least two of the specified angles correctly.

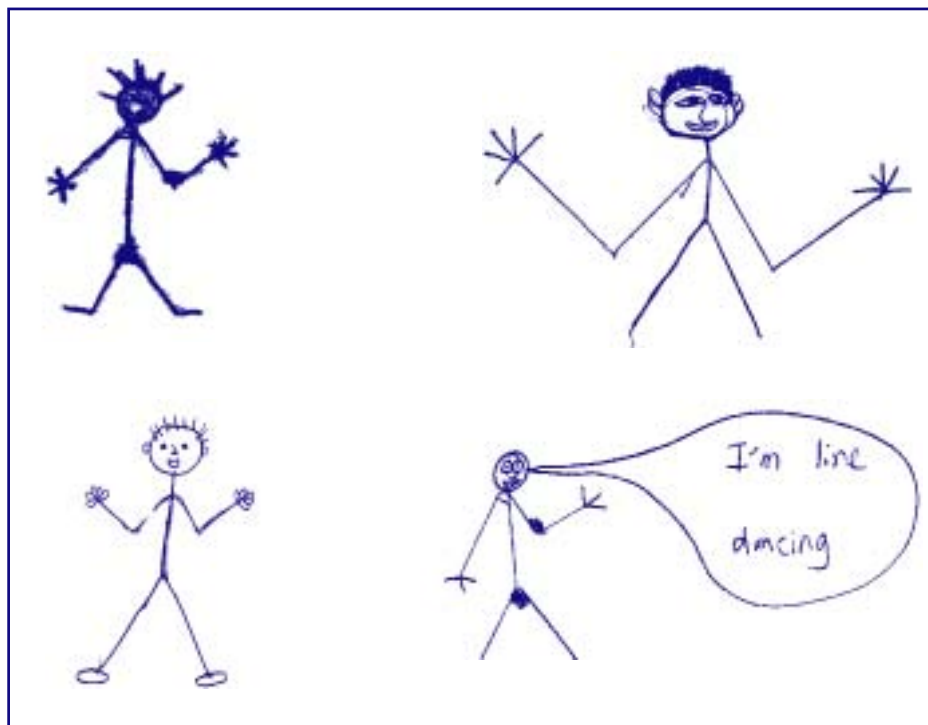


Figure 5: Examples of students' drawings for Lesson 5

## LESSON 6: CLOCKS

Students enjoyed this lesson, in which they made a one-handed clock (using the angle tester for the face and a pencil for the hand) and investigated the angles turned by the hand between various times. They liked using their angle testers and were quite taken with the idea of the link to a clock.

Some teachers were concerned that the students would find 1-line angles extremely difficult. They were surprised at how easily the students seemed to understand the concept and commented that they had found this lesson easier than some earlier lessons on 2-line angles.

In this lesson, the students were given a start and finish time and had to indicate the number of hours and *little angles* from the angle tester and then draw the angle. Most students could find the numbers of hours and little angles correctly, but more than a half had some difficulty with the drawing. Some students drew a picture of a fixed, stereotypical angle (Figure 6a). Other students drew all right angles from 12 o'clock rather than the copying the orientation of the hands (Figure 6b). More than a third of the students showed the orientations of the hands correctly but did not indicate whether the turn was the obtuse or the reflex angle (Figure 6c). Students who did mark the angle often marked the smaller angle. Part of this problem was attributed to the students not having met a reflex angle before this lesson. One teacher had the students write "S" on the starting line and "F" on the finishing line to help them understand clock angles.

(a)

Start	Finish	Hours	Little	Diagram
5	10	5	5	
2	9	7	7	
5	12	7	7	

(b)

Start	Finish	Hours	Little angles	Diagram
1	4	3	3	
3	6	3	3	

(c)

Start	Finish	Hours	Little angles	Diagram
2	9	7	7	
5	12	8	7	

Figure 6: Work samples from Lesson 6

In a second worksheet, students were asked to draw right angles in various orientations. Not all classes completed this worksheet. The right angles were drawn well, but not all students wrote down how they had checked the angle.

## LESSON 7: DOORS

In this lesson, students used a cardboard house façade with two doors to investigate how to interpret the opening of a door in terms of angles. The students enjoyed this lesson, but the majority did not achieve the desired outcome of being able to match the opening of the door to the angle on a pattern block. The most common problems were: (1) not matching one edge of the pattern block to the imaginary line of the closed door and (2) not putting the vertex of the block at the hinge of the door. One teacher drew a chalk line for the imaginary line to indicate the closed position of the door, discussed the angles, and then removed the chalk line.

The flimsiness of the cardboard model was a major problem for this lesson. The house would not stand up, and the doors would not stay in place when opened. Adding extra pieces of cardboard as flaps, and taping the house to the desk, made a difference to the stability of the house but did not improve the doors. The size of the pattern blocks also made demonstrating the angular size of the opening very difficult.

## LESSON 8: SLOPES

Students used sloping rulers to investigate the angle in a slope and then drew these angles. This was a good lesson that fitted the time easily and flowed well from the previous lesson. It was especially useful for emphasising that two lines could have the same angle of slope even though they were of different lengths.

On the worksheet for this lesson, students were given pictures of a sloping ruler. They were asked to draw the angle of slope of each ruler (by adding a horizontal line) and then to draw the angle separately. More than 60% of the students completing this worksheet did so correctly. Some students however, seemed to draw angles at random orientations. A typical example is given in Figure 7, where the student clearly has little idea how to construct the horizontal line to form an angle of slope.

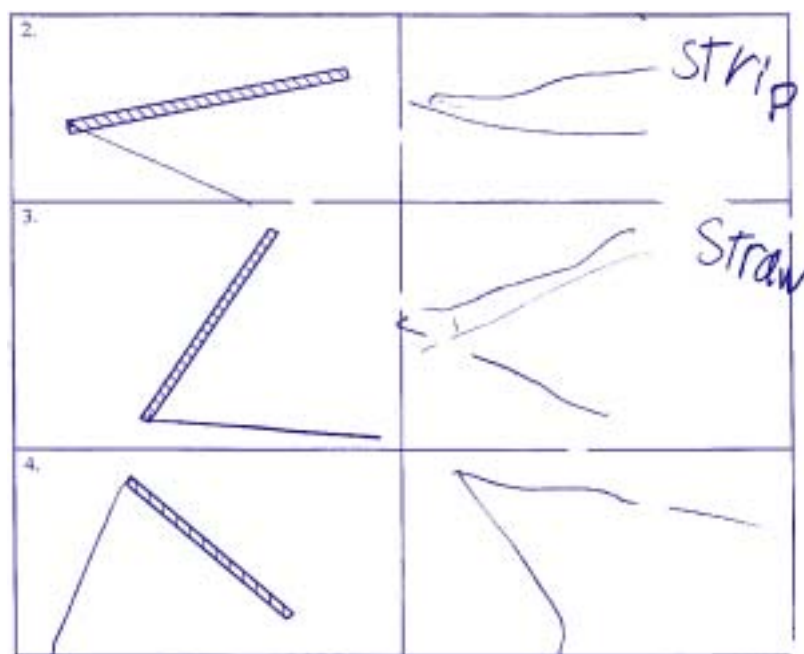


Figure 7: Sample of student work from Lesson 8

Some of the incorrect drawings may be explained by the fact that students found it difficult to transfer the angle of slope from the physical object to the page. One teacher solved this problem by getting the students to draw the angle in the air before drawing it on the page. This lesson also provided an opportunity to use the terms horizontal and vertical.

## LESSON 9: ANGLES, ANGLES EVERYWHERE

Some teachers did not reach this lesson (in which students matched angles in different situations and explained the main features of an angle) because they had run out of time. However, the students who did participate enjoyed the lesson and were particularly taken with the idea of finding half a right angle.

One group of teachers thought that only a few of their students achieved the outcomes for this lesson. Students' major difficulty was in matching angles in different orientations (e.g. a body turn and a sloping ruler). Practical difficulties also arose when students tried to use superimposition and the two objects being matched made this impossible (e.g. a pair of scissors and a door opening).

There was also evidence that some students were still having difficulty abstracting angles from the objects on which they were situated: On the worksheet for this lesson, students were asked to make a single diagram to represent the angle in both of two contexts. The student who made the drawings in Figure 8 did not seem to be able to make this abstraction.






First angle	Second angle	Diagram
The big corner in the blue pattern block	Small door	
Scissors (half open)	Large door	
Green pattern block	Clock starting at 7	
Ruler at any slope	Scissors	
A body turn through half a right angle	A sloping ruler	

Figure 8: Student response to Lesson 9 worksheet

Teachers generally felt that this lesson was too long and could be split into two lessons: one on matching angles and one on describing the general properties of angles.

## LESSON 10: CREATING ANGLES

In this final lesson, it was intended that students should engage in an open-ended task in which they would create some object and then demonstrate the angles in what they have made. Various cross-curricular ideas were suggested. As it turned out, only three classes reached this point. The products were completely different, linked to three different KLAs (Science, Art and HSIE). In each case, all the angles found by the students were 2-line angles.

## SUMMARY EVALUATION

The following summary is based on teachers' responses on a one-page unit evaluation form and general discussion at the second teachers' workshop.

Almost all the teachers agreed that the aims of the unit and the sequence of lessons were appropriate for Year 3 students. The students looked forward to the lessons—one student who was absent from class for a lesson even asked to have a special lesson so he did not miss out.

Asked to identify the best features of the unit, teachers repeatedly mentioned the hands-on nature of the lessons. The use of a variety of materials that were readily available or easily constructed meant that the students actively participated in the lessons. Students were amazed at the diversity of angles; one student said, "I can see angles everywhere". The students could also readily see that angles could be changed, compared and combined. Other comments included the following:

- The activities were sequential and reinforced the concepts.
- Students particularly enjoyed learning the terms associated with the unit: *pivot* and *superimpose* were favourites.
- The use of the pattern blocks and the angle tester as measuring tools was beneficial.

Teachers made many suggestions for improving the unit. The lessons were generally thought to be too long, and some needed to be broken into two lessons. It was thought that some topics could be revisited later in the unit, especially the early lessons where consolidation of the concepts is important. Some teachers felt, however, that spending more lesson time on one concept could be excessive. Other comments included the following:

- Standard angles terminology (e.g. *acute* and *obtuse*) needed to be introduced early in the sequence of lessons.
- Problems with the house, the straw and the small blocks need to be addressed.
- More attention needs to be paid to ESL and STLD students who are in mainstream classes and to those students who finish early.

Teachers thought the unit was good for their professional development. They learned a lot about angles themselves, and they found that the students were capable of "things I never thought they could do".

In summary, the positive aspects of the unit far outweighed any frustrations that were encountered. The sequential nature of the lessons meant that students "were picked up along the way". Having a structured set of lessons for the topic was appealing. Teachers reported that "the students thought it was fun" and felt it was "worth being a part of the project" and they "would definitely use the unit again".

## STUDENT OUTCOMES

Before reporting an analysis of the assessment task results, we consider their validity and reliability.

### VALIDITY OF ASSESSMENT INTERVIEW

The participating teachers felt that four factors affected the validity of the assessment interview.

- The teachers found it difficult to say the sentences in the way specified on the question sheet. They were not used to asking questions such as, “Would you open the scissors to the same angle as that corner?”
- Some of the wording was thought to be confusing, because it was different from that which had been used in the classroom lessons. Also, some terminology was used in the lessons and not in the assessment (e.g. *angle of opening* and *pivot*). It was thought that some students were confused and threatened by the question, “How do you know the angles are the same?” They were more used to being asked, “How can you check the angles are the same?”
- The materials used in the assessment were also different from those used in the lessons. In particular, the angle tester used in the lessons should have been available to the student for testing whether angles were the same. It was felt that many students, who were used to using the angle tester to test whether angles were the same, would have been disadvantaged during the assessment when it was not available.
- Teachers were disappointed that their students did not always show their knowledge in the assessment interview. For example, they felt that students knew the answer to the question “Where is the angle?” but often responded by pointing at the vertex and not indicating the lines.

Teacher felt that the results of the second assessment interview would therefore not accurately reflect exactly how much the students had learned.

### RELIABILITY OF ASSESSMENT PROCEDURES

Because the teachers who participated in the Angles Project were not trained in administering and scoring assessment interviews, it was felt necessary to investigate the reliability of the assessment procedures. To do so, the videotapes submitted by 10 of the 12 teachers were examined. (The videotapes submitted by the other two teachers were, for technical reasons, unusable.) For each teacher, a trained research assistant viewed the videotapes of the assessment interviews of one student and re-scored them without knowing the teacher’s results.

It was found that teachers made three types of mistakes in administering or scoring the assessment interviews:

- Questions were not asked (oversights).
- Responses were not marked on the recording sheet (omissions).
- Responses were not scored correctly (errors).

The relative frequencies of such mistakes can be seen in Table 2.

	FIRST INTERVIEW	SECOND INTERVIEW
Oversights	3%	2%
Omissions	11%	9%
Errors	6%	4%

Table 2: Relative frequency of mistakes in assessment interviews

In addition to the cases reported in Table 2, there were a number of instances where it was impossible to check the student's response (e.g. when the student was pointing to an angle on a small block, or the student or the teacher moved in front of the video).

In the first interview some teachers prompted their students until the correct answer was given, so that the results may be slightly inflated. Prompting was not evident on the second interview. In the third task, some teachers showed their students the right-angled block when they were incorrect; this may have inflated their results on the following task (showing a right angle on a clock face).

## ASSESSMENT RESULTS

Despite reservations as to the validity and reliability of the assessment interviews, it is felt that the assessment data can be cautiously used as a guide to student learning. In making the following analysis, blanks on the recording sheet were presumed to indicate incorrect answers.

*Matching angles.* Figure 9 shows the percentage of students who: (a) were able to correctly match the various angles and (b) used superimposition as their justification.

Between the first and second interviews, there was a substantial increase in the number of students able to correctly match all three pairs of angles. The smaller percentage of students able to match the clock and the slope (on both interviews) may be due to the fact that they are both 1-line angles. An alternative explanation is that some classes did not complete Lesson 9, in which such cross-context matching was examined.

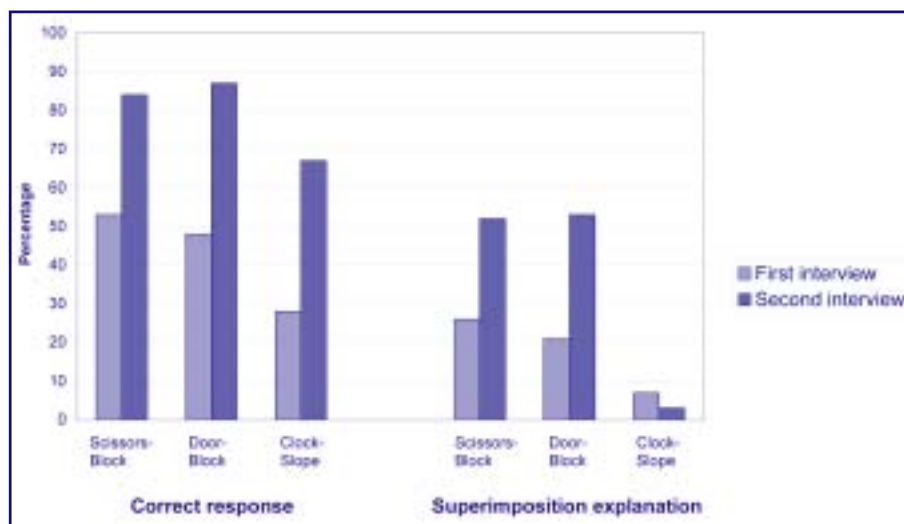


Figure 9: Percentage of sample correctly matching angles and giving superimposition justifications

During the lessons, the students had often used superimposition to check that angles were the same. So it is surprising that, in the second interview, only about half the students checked that the scissors-block and door-block angles were the same by superimposing the block. Using superimposition to check that clock and slope angles are the same is in any case very difficult, if not impossible, unless an intermediary object (block, straw, or angle tester) is used.

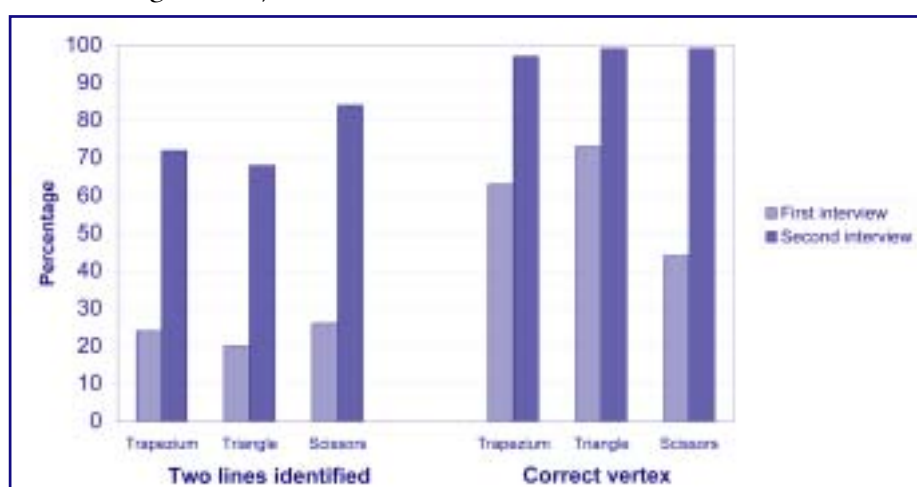


Figure 10: Percentage of sample correctly identifying 2-line angles

*Identifying lines and vertices.* Figure 10 shows the percentage of students who correctly identified the lines and vertices of various 2-line angles. In the first interview, less than a quarter of the students were able to locate the two lines on these angles. After studying the Angles Unit, more than two-thirds could do so. Students found it easiest to identify the scissors angle, but this result may be an artefact of the small size of the blocks. Identifying the vertices of the angles was easier; in the second interview almost all students were able to do this. The large increase in the proportion able to identify the vertex of the scissors angle is particularly remarkable.

Figure 11 shows the percentage of students who correctly identified the various 1-line angles. In the first interview, more than three-quarters of the students saw these angles in terms of a single line and less than a half knew where the vertex was. By the second interview, however, more than a half found the two lines and more than three-quarters found the vertex correctly. While the teachers thought that the students had achieved the lesson objectives in finding angles on a clock, the results shown in Figure 11 suggest that this was the most difficult context.

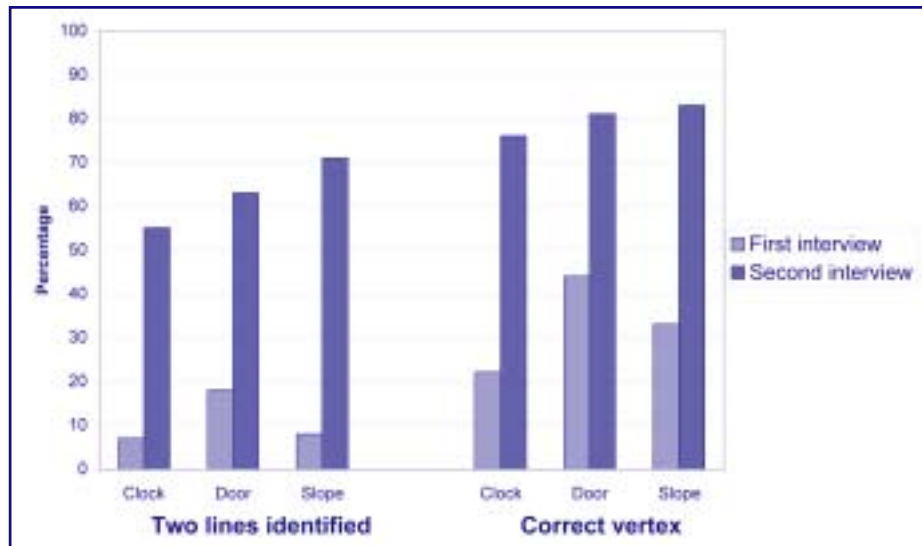


Figure 11: Percentage of sample correctly identifying 1-line angles

Comparing the results shown in Figures 10 and 11 confirms that 1-line angles are more difficult than 2-line angles, even after they have been given special attention.

*Finding angle sizes.* Figure 12 shows the percentage of students who were able to correctly make various specified angles.

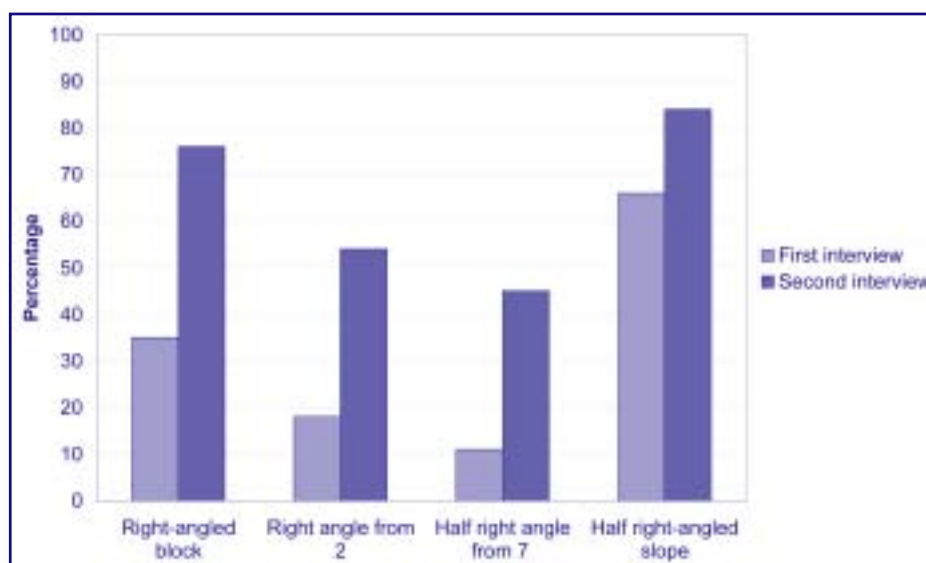


Figure 12: Percentage of sample correctly finding various angle sizes

There are several puzzling aspects of the results for this task:

- The fact that about one-third of the students could identify the pattern block with a right-angled corner in the first interview confirms that many students knew the term *right angle* before starting the Angle Unit. But why were a quarter of the students still unable to find this block after studying the unit?
- Moving the clock hand through a right angle from 2 o'clock was one of the items on the worksheet for Lesson 6. So why were only half the students able to do this on the second interview?
- Why were so many students able to place a ruler at a slope of half a right angle (in both interviews)? The videotapes of the interviews provide an answer to this question: The students had seen the ruler sloping in an earlier question and were sometimes reminded of this by the teacher; and some of the teachers handed the ruler to the students at an angle very close to the correct angle.

## DISCUSSION

### THE ANGLES PROJECT AS PROFESSIONAL DEVELOPMENT

As a professional development exercise, the Angles Project was clearly successful. The teachers all showed a positive response to the ideas put forward and the materials they used, and they undoubtedly learned a great deal about angles from the experience. One teacher summed up the group's reaction when she remarked, "I did not realise that you could teach angles this way". However, various errors teachers made e.g. in scoring students' work suggest that there is still room for improvement in teachers' knowledge.

From the comments teachers made about the Angles Unit, we infer that they also deepened their understanding of several important pedagogical principles—in particular, the value of hands-on materials, links to students' environment, interactive lessons, use of correct terminology, careful sequencing of topics, and continually building on students' previous knowledge.

We are not sure if the teachers learnt anything about assessment, but this was not a major focus of the teacher workshops. On the other hand, judging by the frequency with which Lessons 5 and 9 (the summative lessons on the abstract properties of angles) were omitted, we are fairly sure that most teachers did not learn much about Teaching for Abstraction. Given that this was their first exposure to this method, this finding is perhaps not surprising.

### EFFECTIVENESS OF THE ANGLES UNIT

Teachers judged the Angles Unit to be generally well designed and easy to use. Students clearly found the activities engaging and enjoyable, and the assessment data show that they learned a great deal. However, teachers experienced a number of minor problems and student learning was not uniformly high.

*2-line angles.* At the end of the unit, student understanding of 2-line angles was remarkably high and substantially higher than at the beginning of the unit. Given teachers' concerns as to whether the assessment interviews truly reflected student knowledge, we can be reasonably sure that almost all students achieved the aims of Lessons 1–5.

The problems which students experienced include the following:

- Not enough time for exploration, especially of the pattern blocks (Lesson 1) and the angle tester (Lesson 3).
- The instability of the straw used for transferring angles and the small size of some pattern blocks.
- Locating the vertex of the angle in a pair of scissors at the pivot.

We may conclude that the content of Lessons 1–5 is generally appropriate for Year 3 students.

*1-line angles.* Students' understanding of 1-line angles increased significantly from a very low base, but at the end of the unit only about half of the students appeared to have achieved the aims of Lessons 6–10. The major difficulties which students experienced were as follows:

- Drawing a horizontal line to form an angle of slope.
- Correctly drawing the orientation of the hands of a clock and representing the angle of turn.
- Manipulating the door model.
- Focusing on the abstract angle and ignoring irrelevant physical attributes of the model.

We conclude that the content of Lessons 6–10 are less appropriate for Year 3 students and would be better suited to Year 4 students.

## GENERAL CONCERNS

Time constraints often resulted in teachers omitting or rushing through the summative discussion at the end—the part which focused on the general principles and the more abstract aspects of the lesson. This lack of discussion could account for the students' persistence in focusing on the physical attributes of the model rather than the abstract angle properties.

There is clear evidence that the assessment interviews did not adequately measure student learning. Firstly, the structure of the interviews was in many ways different from what students had experienced in the classroom. Secondly, teachers made frequent errors in scoring student responses.

## IMPLICATIONS

The overall success of the Angles Unit indicates that its content and approach are generally appropriate for incorporation in the *Count Me Into Space* program, but that various revisions are needed before widespread dissemination. In particular, the unit should:

- be spread over a longer time to allow students to become more familiar with the models and engage in more discussion
- provide physical models that are more *user friendly*
- make less use of superimposition and more use of intermediary objects to compare angles
- provide more guidance for the assessment interviews, and these should have a structure closer to that of the classroom activities.

It is suggested that the unit should be expanded and divided into two units, one unit focusing on 2-line angles in Year 3 and one on 1-line angles in Year 4. One possibility for the content of these two units is as follows:

- Year 3:
1. Introduction to pattern blocks
  2. Angles on pattern blocks
  3. Angles in the classroom
  4. Making an angle tester
  5. Comparing corners
  6. Angles in scissors
  7. Angles on the body
  8. Matching angles across contexts

- Year 4:
1. Make shoe box model of house (for use in Lessons 2–5)
  2. Revision of 2-line angles
  3. Slope
  4. Windows
  5. Doors
  6. Clocks
  7. Turns
  8. Matching angles across contexts

Note that the extra time allows the introduction of additional 1-line angle contexts while keeping each unit to a more acceptable length (i.e., 8 rather than 10 lessons). The feedback provided by the teachers in the current project will be of great help in making the many detailed revisions which will be necessary.

## REFERENCES:

- Mitchelmore, M. C. & White, P. (2000). Development of angle concepts by progressive abstraction and generalisation. *Educational Studies in Mathematics*, 41, 209–238.
- Mitchelmore, M. C., & White, P. (2001). *Count me into space: Angles*. NSW Department of Education and Training: Ryde.
- White, P., & Mitchelmore, M. C. (2001). Teaching for Abstraction: Angle as a case in point. In J. Bobis, B. Perry, & M. C. Mitchelmore (Eds.), *Numeracy and beyond* (Proceedings of the 24th annual conference of the Mathematics Education Research Group of Australasia, Sydney, pp. 531–538). Sydney: MERGA.