

Gifted and Talented Education



Extract from *Support package: Curriculum differentiation*

Stage 3

Mathematics: Patterns and algebra



No. 6. Sample unit in Mathematics: Patterns and algebra

Context

The aim of Mathematics K–10 is:

to develop students' mathematical thinking, understanding, competence, and confidence in the application of mathematics, their creativity, enjoyment and appreciation of the subject, and their engagement in lifelong learning.

(Board of Studies, 2002)

The rationale in the syllabus for Years K–10 describes mathematics as a reasoning activity that uses abstraction and generalisation to identify, describe and apply patterns and relationships. It incorporates the processes of questioning, reflecting, reasoning and proof. The process strand called *Working Mathematically* encompasses five overlapping processes:

- questioning
- applying strategies
- communicating
- reasoning
- reflecting.

These processes are integrated into the “learn to” section for each outcome within the content strands. The Working Mathematically strand provides excellent opportunities for enriching and differentiating the mathematical experiences of students. Components of a differentiated Mathematics curriculum include:

- the development of spatial skills through geometry and other media, for example, exploring fractals, platonic solids, geodesic domes
- a focus on problem-solving skills that extend an original problem and explore different solutions
- an emphasis on the use of technology
- logic problems that require deductive thinking skills and inference
- algebraic manipulations arising from an exploration of patterns
- statistics and probability.

Outcomes

NS3.1

- *Orders, reads and writes numbers of any size*

WMS3.1

- *Asks questions that could be explored using mathematics in relation to Stage 3 content*

WMS3.2

- *Selects and applies appropriate problem-solving strategies, including technological applications, in undertaking investigations*

Table 17. Example of lesson plan in Stage 3 Mathematics

The content of this unit is covered under **Additional content** (p. 10) in the syllabus.

Objective	Activity	Outcomes
Students apply an understanding of place value and the role of zero to read, write and order numbers of any size.	Students recognise the place value of numbers in numerical data that can be found in the media, for example, house prices, salaries paid to executives.	Students can interpret, compare and contrast information from the Internet, media, environment and other sources.
	Students investigate: <ul style="list-style-type: none"> • numbers in different bases e.g. clock to base 2 • large numbers e.g. googol and googolplex • the number of squares on a chess board (King Kaid of India) • the number of eyedrops of water that will fit on a ten cent coin. 	Students can apply strategies to estimate large quantities.
	The following activities could be allocated using work cards. A list of resources to support these activities is included at the end of this sample unit. To expand the content of NS3.1, in small groups, students could investigate some of the following concepts: <ol style="list-style-type: none"> 1. to extend thinking on infinity <ol style="list-style-type: none"> (a) draw pictures of infinity e.g. a person reading a book which has a picture on the cover of a person reading a book which has, or use two mirrors (the teacher could provide samples of similar artwork) (b) make and study Moebius strips, Klein bottles and Klein cubes (c) class writes an infinite poem e.g. circular (d) using isometric dot paper and centimetre cubes (after work on visualising and 3D shapes), the challenge is to build something that is impossible to draw or draw something that is impossible to build e.g. an impossible cube. Drawings can change perspective and work towards infinity (e) look at the work of M.C. Escher and the way it relates to infinity. 2. to explore numbers <ol style="list-style-type: none"> (a) perfect numbers, happy numbers, prime numbers, amicable numbers, palindromic numbers, Fibonnaci numbers, numbers in Pascal's triangle (b) by adding numbers 1–100 (hint, begin with adding the numbers 1–10; what patterns can you find?) (c) research the work of Carl Gauss Fold different sized pieces of paper in halves. What patterns can you find in the number of times a piece of paper may be folded in half? What relationship does this have with infinity? 	

The exploration of infinity provides an extension activity for gifted students because it caters to their ability to deal with difficult abstract concepts. These activities illustrate how mathematical concepts can be explored using interdisciplinary approaches. The following table provides some useful question cues to stimulate imaginative and creative solutions to mathematical problems.

Table 18. Question hierarchy to guide investigations

Levels of thinking	Guide thinking
Knowledge: recalls or memorises information	What have we been working on that might help with this investigation?
Translations: changes information into another form	How could you write/draw what you are doing? Is there a way to record what you've found that might help us see more patterns?
Interpretation: discovers relationships	What's the same? What's different? Can you group these in some way? Can you see a pattern?
Application: uses appropriate generalisations and skill to solve problems	How can this pattern help you find an answer? What do you think comes next? Why? What can you say about this type of problem? What mathematics could be used?
Analysis: solves a problem—conscious knowledge of the thinking, examines interrelationships of parts	What have you discovered? How did you find that out? Why do you think that? What made you decide to do it in that way?
Synthesis: solves a problem that requires original, creative thinking, goes beyond the original problem, considers alternatives	Who has a different solution? Are everybody's results the same? Why/Why not? What would happen if ...?
Evaluation: makes a judgement, considers standards and quality of solutions	Have we found all the possibilities? How do we know? Have you thought of another way this could be done? Do you think we have found the best solution?

(Way, 2002)

This unit was originally created by Beecroft Public School.

Resources

- Erickson, T. (1989). *Get it together: Math problems for groups Grades 4–12*. Berkley, Calif.: Equals, University of California.
- Finlay, E. & Lowe, I. (1993). *Chance and data: Exploring real data*. Carlton, Vic.: Curriculum Corporation.
- Lovitt, C., Clarke, D. & Mathematics Curriculum and Teaching Program (Australia). (1988). *Activity bank. Volume I*. Canberra: Curriculum Development Centre.
- Lovitt, C., Clarke, D. & Mathematics Curriculum and Teaching Program (Australia). (1988). *Activity bank. Volume II*. Canberra: Curriculum Development Centre.

Electronic

The following resources were available 9 November 2004:

Intellitech Systems Inc. *Binary clock*.

http://www.delphiforfun.org/Programs/binary_clock.htm

Lanius, C. (Rice University). *Fractals*.

<http://math.rice.edu/~lanius/frac/>

About Inc. *Geodesic domes*.

<http://architecture.about.com/library/ucdome.htm>

Gifted Education Services. *Fermi off the wall math league*.

<http://www.int287.k12.mn.us/gifted/fermiarchive.html>

Heng, O.K. *The perfect number journey*.

<http://home1.pacific.net.sg/~novelway/MEW2/lesson1.html>

Mathematical Association of New South Wales Inc.

<http://hsc.csu.edu.au/pta/mansw/>

Tanunda Lutheran School. *King Kaid of India*.

http://www.tls.sa.edu.au/Yearlevels/Mr%20Emmett/Year_7.htm

University of Exeter, Centre for innovation in mathematics teaching. *Klein cube*.

<http://www.ex.ac.uk/cimt/mepres/allgcse/bs7act1.pdf>

University of North Carolina. *The oldest Escher collection on the web*.

<http://home.comcast.net/~davemc0/Escher>