Characterising Numeracy in the middle years

3.1 What does the literature say about the Contexts and Meanings of Numeracy:

Public Policy context:

In the 1980's France estimated that around 30% of its education budget covered costs arising from failure at school. One of the reasons for numeracy being spotlighted in public policy across the world is the high cost that low levels of achievement cause both within the education budgets and in terms of later dependence on the state due to unemployment or low-paid spasmodic employment. Once failure was understood to be inevitable for some students. This is no longer the case.

The International Association for the Evaluation of Educational Achievement's (IEA), *Third International Mathematics and Science Study* (TIMSS) of 13-year-olds has shown that there is up to a four-year gap in achievement between the highest and lowest achieving students in Mathematics.[1] For Australia, TIMSS indicated that this age group was outperformed by eight countries (eg., Singapore, Korea, Japan and Hong Kong) on par with Netherlands, France and Canada and better than other English-speaking countries including the USA.

Increasingly, however, it appears to be the view of policy-makers that the statutory requirements of approximately nine compulsory years of education are inadequate to meet the requirements of daily life in the 21st century and that the acquisition of numeracy is much more crucial than in the past.

Psychological context:

Numeracy has most frequently been situated within a psychological theory of learning. Developmental models such as that of Piaget have been pervasive during the past twenty-thirty years in mathematics classrooms. More recently notions of constructivism have been influential within this paradigm. Numeracy is generally seen to be a capacity for cognitive conceptualising; potential achievement appears to most often be viewed in relation to innate ability. Disposition and self-esteem rate highly as factors which might contribute to numeracy success.

Economic context:

The OECD suggests that "quantitative literacy" appears to correlate with income and this is more often the case in "open economies" like Canada and the US. Economic inequality has tended to rise over the past two decades, despite large investments in education. Comparing TIMSS data with the IALS data, it is reported that

"in every case 8th grade students who had at least one parent who completed some tertiary education had higher average scores in mathematics than students whose parents had received less education.[2]

The *Programme for International Student Assessment* (PISA) longitudinal study of 15-year-olds' commencing in 2000 assesses "Mathematical Literacy" defining it as:

"Mathematics literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded mathematical judgments and to engage in mathematics, in ways that meet the needs of that individual's current and future life as a constructive, concerned and reflective citizen."[3]

Social/cultural/political context:

Unlike literacy, numeracy has been less actively defined and debated in this context. Increasingly, however, numeracy is emerging as an aspect of literacy, “quantitative, document”, “mathematical”, in the international literature. The increasing prevalence of multimodal texts require the reader to possess multiliteracies. Many of these demand knowledge of mathematical values and processes simultaneously with other literacy requirements. Technology allows the presentation of mathematical information in numerous formats requiring a range of concurrently applied literacies and numeracies. Visual literacy is increasingly a crucial component of numeracy. Knowledge of a ‘grammar’ of images and a ‘grammar’ of the discourse of mathematics are increasingly suggested to be required as explicitly taught components of the curriculum. Questions of agency in the pedagogical process and reader-positioning in mathematics are beginning to arise in the literature leading to a critical interrogation of traditional explanations of mathematical success. From a socially critical perspective Danish Skovsmose writes about his rejection of Piagetian concretisation:

“I prefer the idea of mathematisation which I understand to be the activity of finding systems and regularities in a chaotic daily life situation.” [4]

Institutional context:

The Commonwealth’s outline of numeracy policies for schools, *Numeracy, a Priority for All: Challenges for Australian Schools* [5] compares the UK definition of numeracy with the broader notion of Australia:

UK National Numeracy Task Force (1998):

..an understanding of the number system, a repertoire of computational skills and an inclination and ability to solve number problems in a variety of contexts. Numeracy also demands practical understanding of the ways in which information is gathered by counting and measuring, and is presented in graphs, diagrams, charts and tables.

Australian Association of Mathematics Teachers (1997):

..to be numerate is to use mathematics effectively to meet the general demands of life at home, in paid work, and for participation in community and civic life.

In school education, numeracy is a fundamental component of learning, performance, discourse and critique across all areas of the curriculum. It involves the disposition to use, in context, a combination of:

- underpinning mathematical concepts and skills from across the discipline (numerical, spatial, graphical, statistical and algebraic;
- mathematical thinking and strategies,
- general thinking skills; and
- grounded appreciation of context.

This Commonwealth document cites Hill and Russell noting that during the middle years there is evidence of a plateau or decline in students’ learning and acknowledges the work of the Australian Curriculum Studies Association concerning the potential for lost motivation or disengagement during these years.

Apart from ensuring that there are effective linkages between primary and secondary schools, student-centred approaches, clear specification of content, in-depth learning, emphasis on thinking, problem-solving, autonomous learning and challenging students are mentioned as appropriate tasks for the middle years.

Projects currently in operation are cited, focusing on middle years numeracy: *Junior Secondary Numeracy Project* in South Australia: a case study approach to school-based action-research, the *Transition Numeracy Project in Western Australia*: professional development program helping teachers to identify students at risk at the transition stage, *Planning and Teaching for Numeracy in Years 7-9* in Tasmania: focused on identifying pedagogical practices and developing materials.

A further project focusing on pedagogy is *Numeracy Plus* in NSW - building on *the Count Me In Too* program of video assessment of student learning in order to both identify how it is that children apply numeracy understandings and to help teachers learn to design effective programs for students experiencing difficulties with numeracy.

3.2 What does the literature say about High Expectations and Numeracy?

One of the difficulties with expectations for numeracy learning concerns knowing what numeracy goals might be. Defining the boundaries of numeracy and expectations for learning remains illusory, complicated by the question of its location. Views vary regarding whether numeracy development is located within the field of mathematics, either as a component of the field, or beyond - though dependent on - mathematics, as an umbrella. This question is further complicated by increasing evidence of a view of numeracy as a form of literacy.

The OECD final report on Adult Literacy shows patterns of achievement for 16-65 year-olds in three domains of literacy, all of which are relevant to numeracy to some degree; the third - quantitative - being specifically arithmetic:

Prose Literacy: the knowledge and skills needed to understand and use information from texts including editorials, news stories, brochures and instruction manuals.

Document Literacy: the knowledge and skills required to locate and use information contained in various formats, including job applications, payroll forms, transportation schedules, maps, tables and charts.

Quantitative Literacy: the knowledge and skills required to apply arithmetic operations, either alone of sequentially, to numbers embedded in printed materials, such as balancing a chequebook, figuring out a tip, completing an order form or determining the amount of interest on a loan from an advertisement."[6]

Curriculum Support for the teaching of Mathematics 7-12 [7]contests this colonisation of numeracy by literacy:

> "If we were to define document numeracy it may well subsume document literacy. As well as locating and using information contained in tables, charts and graphs, document numeracy would incorporate the ability to select appropriate formats to represent information and construct graphs and tables. Adding a dimension of critical understanding to document numeracy would also require students to identify graphs and data that are misleading...
>
> Prose numeracy includes the translation of information from prose to diagrammatic or symbolic formats. As a specific numeracy skill, this area of translation needs far more emphasis than it currently receives in classes. Prose numeracy needs to take into account the range of text types which students encounter."

Citing the work of Richard Skemp in the seventies, *Developing Understanding* (2000)[8] defines two types of mathematical understanding. Perhaps these represent the crucial distinction between maths-in-school and numeracy:

Relational knowing what to do and why

Instrumental rules without reasons

The author asks the question: "Are we happy if year 7 students know the rule and are able to apply it?" and suggests that a student's capacity to replay procedures often passes for understanding.

"We usually assess the procedures not the understanding. Understanding the concept is often reserved for only the most able students"

A further question posed by Skemp is: "What can we do when we understand what we can't do when we don't understand?

The author suggests, here, that we can use knowledge flexibly; we can depart from the procedural script, reverse the process if necessary or make connections to other areas.

Spillane discusses his case study of a Grade 5 teacher in the US which compares the efforts of the teacher in reforming her practice in both literacy and numeracy according to directions suggested by district consultants. He observes that

"subject-matter-specific differences in the teacher's identity as a teacher and as a learner influenced the very different learning opportunities, as well as the substantial differences in her stance as a learner, in mathematics compared with literacy...with mathematics Adams felt insecure, lacking the confidence to take risks in her classroom and explore for herself different approaches...her experience with mathematics in school centered on memorising rules and procedures.." I guess I never liked math because it had all these right answers and it didn't allow for divergent thinking...I was a typical female when it came to math."[9]

In this work, Spillane cites the National Council of Teachers of Mathematics (US 1989) regarding what it means to be 'mathematically literate':

"having the ability to reason and investigate mathematically - to frame and solve mathematical problems, to offer conjectures, to develop arguments in support of their conjectures, to communicate with others about mathematical ideas."

Spillane elaborates ...classroom norms and discourse patterns should reflect that neither the teacher nor the text is the final arbitrator of valid interpretations - students can and should be legitimate constructors of meaning."

Klein[10] argues that, despite the rhetoric regarding numeracy:

"my sense of classroom mathematics is that in reality nothing but the packaging has changed - the children of the 21st century are not authorised to take themselves up as (becoming) numerate agents."

Klein explains the ways in which individuals are constituted through discourses; positioned within discourses and actively involved in them. She indicates that, according to this model, students are discursively produced as numerate individuals or not within the classroom operation of the discourse:

" This notion of constituted subjectivity is particularly relevant to mathematics teaching and learning because it makes very clear that what teachers do and say in classrooms, and how they do and say it, is implicated in what students come to know of mathematics and of themselves as numerate individuals....to be seen to have agency of any kind, including in one's own eyes, an individual needs to be competent in term of the uses of language and recognised practices of the discourse, and must be positioned by others as one who has legitimate authority to speak and be heard."

Further, Klein critiques psychological - developmental and constructivist - models of learning:

"...are based on humanistic understandings of an individual who can freely choose to be competent and act rationally...these discourses set up and maintain story lines that, when acted upon by teachers and the learners themselves, often marginalise...for example the discourse of 'natural development' has resulted in teachers standing back, for example in problem solving activities, where they should more appropriately intervene and teach the concepts and skills on which the problem is based."[11]

The issue of age-based expectations and progress, regardless of class, gender or culture, is raised as problematic; resulting from the developmental psychology framework the distinctive binaries right/wrong, capable/slow sort and 'position' the student mathematical agent.

"..for it is only in doing the mathematics, in being supported and recognised as displaying numerate behaviour in the classroom, that they come to taste the agency that will stay with them (as part of their constituted subjectivity) forever...After all, the teacher's task is to teach in such a way that the students can fly alone."[12]

The critical question appears to be, "Which numeracy?" It is difficult to determine expectations when the boundaries within which those expectations are located is blurred.

Willis asks this question. She offers three versions of what numeracy might mean:

1. numeracy synonymous with mathematics; the basic skills notion which has a determination to increase students mathematical repertoire,
2. numeracy as communicative competence; context specific skills and knowledge which includes a goal to increase the repertoire of situations in which students can operate mathematically,
3. numeracy which bridges the gap between mathematics and the real world; the capacity to analyse situations from a mathematical perspective requiring the development of students' strategic repertoire.

Willis suggests, however, that whichever view we might take,

"it is clear that what we do on behalf of numeracy in schools largely is to teach mathematics...increasingly this position is seen to be unsatisfactory."[13]

Teese et al writing about gender factors suggest

"Girls and boys do not use secondary school to the same extent or in the same ways. The full-time labour market for young people favours boys, who thus rely less on school. Girls are required to make more intensive use of school because their non-school work and training options are more limited. In this sense they rely more on school than boys."[14]

They remind us that large sections of the curriculum continue to indicate gender imbalance.

"Boys are poorly represented in the humanities and the arts. Specialist maths and physical science subjects have disproportionate numbers of male candidates."[15]

Furthermore they argue under-enrolment in maths in the senior years is a "concealed form of disadvantage":

"Girls experience a performance advantage through relegation - that is, they are more likely than boys to select maths options below their levels of ability and thus appear to outperform boys. But this is achieved at the cost of studying at a lower level of the maths curriculum....many boys take on maths above their ability levels and fail accordingly....it is only in the regions of high socio-economic status that girls' participation reaches or exceeds even the average level for boys. And working class girls, while enrolling less in maths than any other group, have higher probabilities of failure."[16]

In Howard's study of views of Aboriginal mathematical learning one of the non-Aboriginal teachers comments:

"You don't get many Aboriginal kids in the top class...there's a lot of that self-fulfilling prophecy where people expect people to perform in certain ways. Perhaps you frame things so it works out how you expect it to work out."[17]

Strategies for students experiencing difficulties in numeracy across Australia, are studied by Milton[18]. She cites the views of Ginsberg in the US who suggests that numeracy learning difficulties are socially constructed resulting from a discomfort with maths of elementary teachers, poor teaching skills, poor schools and low expectations of some groups of students.

In considering the issue of ‘real-life’ maths in school, Milton observes that few of the case study schools reported trying to make maths tasks relevant to real life situations by explaining how they could be used outside school. Further, she comments that it was clear that teachers spend more time and emphasis on skills and facts rather than understanding.
The nature of assessment tools in mathematics is questioned relative to their value as an indicator of numeracy.

The identification of students experiencing difficulty with numeracy is not clear-cut nor as well practised as identification of literacy difficulties. In the later years of primary school identification is hampered, Milton suggests, by lack of suitable assessment material, confusing numeracy with literacy problems and acceptance that some children cannot do maths. It seems from her case studies and surveys that the most common form of assessment of numeracy is the statewide assessment test.

In the case study schools a majority of principals indicated that there were between 10-30% of primary students experiencing difficulties with Maths.

van Kraayenoord[19] et al’s study of numeracy for students with disabilities indicated a dearth of literature in this field.

“there was a lack of published literature in the area of numeracy development for students with all types of disabilities and what research we found was often dated....This study did indicate, however, that where teachers and parents had high expectations for developing independence at school and at home these students did well.”

The study of students with disabilities suggests that these students represent 3-5% of the school population. A concern was raised that:

”In Australia we know little about the interface between disability and socio-economic disadvantage or about the interface between disability and cultural and linguistic diversity.”

Although much of the literature indicates variable expectations with regard to numeracy. The question of, and strategies for, raising expectations for all is infrequently evident.

3.3 What does the literature say about the needs of Adolescent Learners and Numeracy?

"Humanist discourses have for many years stressed the importance of the students' confidence and positive disposition if they were to be considered numerate in the world beyond school. It was as if these are personal attributes that any rational and responsible individual could nurture if only one applied oneself assiduously enough to learning the mathematics. An alternative poststructuralist understanding of the individual constituted through discourses places the onus squarely on the uses of language and practices of school mathematics to construct students who know themselves as competent and capable numerate persons."[20]

The recent evaluation of a pilot Numeracy program[21] aimed at year 7 students experiencing least success in NSW shows that focusing teachers' attention directly (via video-footage in this case) on how it is that students go about resolving numeracy questions is a powerful teacher learning tool.

"I have learned more from doing these assessments than anything else in my years as a mathematics teacher. It really opens your eyes to the way children think and develop problems in learning mathematics," comments one of the teachers involved.

Results from the TIMSS mathematics survey indicate little gender differential in performance amongst the adolescent group. Students from non-English-speaking background students who spoke English at home performed at a higher level than those who did not and results for Indigenous students were significantly lower than for non-indigenous students.

A parent reflects on her daughter's experience of the middle years considering why she was voting with her feet:[22]

"I have thought long and hard about what there is about a school system that can engage a child's interest so little that it is better to spend the day doped out alone in her own bedroom than to go to school to learn or to socialise with her friends...

What could the middle years have offered Elizabeth? She tells me that what she really wanted and didn't get was intellectual challenge. This may sound strange from a fairly errant teenager. But as she puts it:

I got to meet lots of people at school and the social life is sick (this means fantastic in the lingo of the age) *but I didn't need school for that. I've got lots of friends outside school and I see them all the time. The teachers didn't really care if I was interested in the work. All they cared about is that I did it. And when it was so boring why would I bother! The subjects are boring. I hate maths, what is the use of learning it anyway? English was good when we talked about real issues, but the assignments were really easy. If I did them it took me no time at all. None of my subjects really made me think I'd like to know more about this, or I'd like to explore that idea. Except for perhaps drama where I really got involved and I did go to those classes."*

Keast[23] recounts a school-based study at Murtoa Secondary College in rural Victoria of year 8 students in which the three mathematics classes were reorganised as 1 x girls only, 1 x boys only and 1 x mixed gender in order to know whether differences in both confidence and achievement resulted from gender-specific groupings. It was found that little difference was made in confidence levels of students. But noticeable differences resulted in achievement for both boys and girls in the single-sex classes. Parents of girls had been enthusiastic whereas boys' parents were less supportive. It was found that it was the boys' outcomes which were the most noticeably improved.

Howard reports on an ethnographic study undertaken in rural NSW which investigated the beliefs of Aboriginal students, parents and educators towards the learning and teaching of mathematics in the final two years of primary school.

The students indicated clearly that they were dissatisfied with a maths curriculum which relied on sheets and stencils. Sam is quoted,

"Get us all to sit around and have a good talk..instead of just giving out sheets all the time."

Success in maths is associated with getting things right. Parents were concerned about issues of institutional racism, of artificial age barriers for learning expectations when communities do not restrict access and participation along age grounds, the perception that discipline had become more important than learning and, it appeared to them, that schools focused on literacy with not the same degree of interest or funding in numeracy.

The Aboriginal educators felt that the students see maths as boring. They explained that the children are thinking in two different systems -

"when children want to ask questions they don't think about how to phrase the questions in white terms ..then they get picked up on how they ask the question, not what they want to know."[24]

The Aboriginal educators also indicate that, by this age, the children have realised that "life is different for them". They explain that education separates; "it changes the kids from the way they do things at home". This becomes a dilemma for all involved. Finally, it is suggested that many teachers have limited expectations of the potential in Maths of Aboriginal students.

Klein appeals to teachers:

"For every lesson, for each student the teacher needs to think about

a) what mathematics was learned today and how relevant was it to the world of tomorrow?

b) to what extent was each student able to establish her/himself as a numerate individual?"[25]

Barratt reports on South Australian schools' progress towards responding to the specific needs of young adolescents. At Seaford, a years 6- 12 school, students are grouped in 'sub-schools'

"where they get to know each other and their team of teachers well...In the sub-schools the mapping of curriculum and use of benchmarks to graduate are strong elements and are reflected in team planning units of work...there is a sense in which the students can't run and hide in these close knit and connected communities."[26]

Barratt reports that the new South *Australian Curriculum Standards and Accountability Framework* has created bands for learning including a Middle Years band acknowledging the need to "conceive of, construct and deliver curriculum differently for these students".

3.4 What does the literature say about Explicit and Systematic Teaching of Numeracy?

The TIMSS data not only included student assessments but detailed analysis of classroom practices in Japan, Germany and USA for eighth graders. Peter Gould[27] citing the work of Stigler and Heibert (1999) highlights some of the findings:

- teaching methods are the critical factor in determining mathematics performance of different countries,
- the quality of mathematics varied considerably across the 15 geometry and 15 algebra lessons randomly selected. Analysts had no knowledge of country in which the lesson had been taught and found for the USA that 89% of lessons were considered to be of low content, 11% medium and nil high. For Japan, in contrast, where student outcomes were of a high standard, 11% of lessons demonstrated low content levels, 51% medium and 39% high levels of content,
- the purposes for use of visual aids in classrooms varied. In USA the use of the overhead projector was common. Teachers made only the information currently in focus visible to the students. In Japan, the chalkboard was most frequently used. Systematically the lesson was transcribed visually from left to right across the board, allowing students to make links between old and new information. It is argued that these methods suggest differing purposes; in the USA, for control of student focus and, in Japan, for supporting students to make connections in their knowledge.

Teachers participating in the NSW Pilot Project *Numeracy Plus* expressed a lack of confidence in their knowledge of ‘basic’ numeracy:

“The overarching concern expressed by participating teachers was that they had little or no understanding of the explicit analysis of elementary counting, arithmetic, place value, base ten knowledge and multiplication and division processes.”

...“my priority is motivation - they need success. You need to bypass to get correct answers. We just do it - you don’t need to explain...you get self-esteem first and then you go back.”

The evaluation found that although teachers became more aware of inefficient numeracy strategies used by students, the basic numeracy teaching strategies became, more often, an adjunct to normal classroom lessons.

“there was an underlying belief that the syllabus and use of an appropriate textbook was a requirement - despite the level of ability of the student.” [28]

Further, one of the main factors impacting on the success of the pilot of *Numeracy Plus* was found to be the “structure of secondary schools” in relation to timetabling, staffing and streaming of students.

Following amalgamation of a group of schools in South Australia which established a middle school - Norwood Morialta High School - the middle-school methodology committee researched boys’ and girls’ attitudes to learning & teaching:

“What emerged conclusively was both sexes’ need for more say in the curriculum and a very strong desire for more interaction and engagement with staff... year 10 boys are the most at risk of leaving school early. More of them feel alienated from school than do any other group, and it behoves us as teachers to grapple with boys’ alienation from the curriculum.”[29]

In Victoria, Morialta Senior College's study of single-sex mathematics classes in Year 8 considered Gilligan's work on ways of knowing applying the theory of 'separate-knower' and 'connected-knower' to the boys' and girls' classes respectively.

"Separate knowers appear to learn by formal, structured and explicit instruction which was what was observed in the single-sex boys class whilst Connected-knowers learn by empathy and relating to others, the cooperation and sharing of ideas and knowledge that was observed in the single-sex girls class."[30]

Morony reports on the *Indigenous Students and Numeracy* [31] (ISAN) project which during one year supported a number of schools to undertake school-based approaches improve numeracy achievement. In this short time he suggests that. although the approaches applied were diverse the common themes which were found to make a difference were:

- providing time for teachers to attend to students' numeracy development as a special responsibility,
- explicit and extensive involvement of para-professionals and parents,
- community involvement in decisions about teaching strategies,
- special attention to English language of mathematics.

Klein argues that it appears that in the past has been

"a conspiracy to conceal from students all the exciting and challenging aspects of mathematics."[32]

Veel[33] provides a comprehensive, if tentative, analysis of school mathematics discourse as a base for arguing against the appeal of democratising mathematics learning by "weakened framing" of the mathematics classrooms in the form of negotiated, small-group, collaborative learning opening up alternative interpretations. Recognising that strong framing has frequently caused mathematical pedagogy to be oppressive, he suggests a "more responsible approach" might be that which allows for "waves of weak and strong framing" such as that recommended in the Teaching/learning cycle for literacy such that there is room for student voice and negotiation at times and careful scaffolding and guidance at other times.

Veel's work drawing on secondary mathematics classrooms in inner-urban disadvantaged communities appears to be quite unique in providing a description of the key techniques by which mathematical discourse is practised.
He notes:

- the predominance of teacher spoken language,
- the predominance of distinctive patterns of spoken language interaction,
- the technical fields of knowledge construed through spoken and written language including technical lexis, grammatical metaphor, relational clauses and nominal groups,
- the hierarchical ordering of mathematical concepts through language and,
- the gap between student use of mathematical language and teacher/textbook use of mathematical language.

This work not only makes visible the ways in which the technicality of mathematics is constructed in order that teachers might be enabled to teach with greater explicitness but it sheds light on considerations of pedagogy and ways of more effectively distributing 'knower' roles between teacher and students.

Milton's case studies highlighted some schools with key focuses on numeracy. At Oceanview school (WA)

"teaching of numeracy revolves around the students' use of journals in which they record before- and after-lesson responses to questions such as, 'What is a fraction?' Lessons have a practical base and there is much use of concrete materials to solve practical problems by students often in groups. The lessons are characterised by explicit 'think alouds' by the teacher in which the processes for calculating algorithms or solving problems are made transparent. Children are also encouraged to verbalise the process that they have used in performing mathematical tasks. In this way teachers can make internal processes explicit by asking the child, "How did you work that out?" In this way the teacher is able to identify faulty reasoning or calculating. There is a strong emphasis in showing students when and where they would find school-based tasks useful in the outside world and after they have left school."[34]

Metacognitive processes such as those outlined here are viewed as the most effective interventions for children experiencing difficulty with numeracy.

Goos' study with senior students investigates patterns of collaborative metacognitive activity considering to what extent peer interaction mediates cognitive activity. Her conclusions at this stage suggest:

"it would be misleading to claim peer collaboration always achieves a mathematically productive outcome. ...in these circumstances collaboration was cognitively fruitless because the students did not have access to the means for resolving their uncertainty."[35]

The three factors proposed as responsible are: lack of prior experience, no empirical basis for evaluating the sense of their answers and lack of access to knowledgeable others.

Rather than focusing on arithmetic calculations and repetition drills mathematics at Portsmouth Middle School in US is an "exploration using visual and hands-on methods." When teams of students

"...devise solutions to problems they demonstrate their own mathematical reasoning by illustrating their team's solution at the overhead projector. ..teachers make a point of not asking students to state the correct answer. Instead they constantly request that students communicate the process..."

From the teacher's point of view:

"It is my job to help them understand that disequilibrium is all right. It means that they are struggling and that, in reality, confusion is a condition for real learning."[36]

van Kraayenoord's study of students with disabilities raises concern that many students with disabilities rely on teacher aides for much literacy and numeracy support. In most cases these people have received little or no training.

3.5 What does the literature say about Subject-Specific Numeracy?

Lo Bianco and Freebody suggest that each subject, through its disciplines and traditions presents an orientation to knowledge using particular written, spoken and symbolic forms:
"There are generic and systematic differences in the functions that written texts serve, differences in their authorised and approved ways of presenting, explaining and debating information and its implications. Other differences relate to particular grammatical features. Finally, important differences in ways of non-verbal accompaniments to language such as numerical, graphical and pictorial representations of information."[37]

Numeracy for All cites *The Strategic Review of Mathematical Sciences and advanced Mathematical Services in Australia* (1996)[38] indicating the high degree of pervasiveness of mathematics throughout our 'knowledge-based' society, listing medicine, commerce, industry, life and physical sciences, social sciences, engineering and technology as those requiring numeracy.

Landman discusses the mathematicisation of industry regarding the role of mathematical modelling in the design and problem-solving process. She says that multiple approaches to the same problem in industry can be made. The ability to use cross-thinking across disciplines - seeing analogies between different fields - can lead to innovation and invention.
"The rapid development of computer hardware and software during the past years has made it possible for computer simulations and visualisation to replace experiments and the construction of prototypes in product design, thereby reducing costs and development times."[39]

Skovsmose argues comprehensively that a social process is currently at work which is formatting our society mathematically. Mathematics is colonising part of our reality and re-ordering it.
"Applications of mathematics have shown growth of epidemic proportions. Previously, subjects like physics, astronomy and chemistry were the main areas of application of mathematics, but now no subject area seems to be immune from quantitative analysis. The escalation of the applications of mathematics is closely connected to the development of information technology, making it possible to handle complicated mathematical models and overwhelming quantities of data. Information technology may be seen as the materialisation of mathematics." [40]
The conceptualisation of technology here is a duality of 'machine' and 'competence'.

A description of a thematic approach to mathematical education in a Danish classroom for 10-11 year olds is provided as a means of exemplifying the potential for 'critical numeracy'.
"...mathematics has normally been conceived of as a stiff and formal subject with a high resistance towards interdisciplinarity and cooperation, but also a subject not essential to the establishment of a critical education."

The 'first wave' of critical mathematics as exemplified in the classroom program focused on Economic Relationships and concerns:

- critical competence — the subject matter cannot be taken for granted on the basis of established tradition but must be continually subject to revision,
- critical distance — from the curriculum
- critical engagement — in common educational and social endeavours

The conditions for design of the Unit of Work were:

1. The topic has to be well-known to the children or it must be possible to describe it in non-mathematical terms
2. It must be possible for the children to enter at different levels,
3. The theme must possess a value of its own,
4. Working with the theme must generate mathematical concepts, ideas about systematisation or ideas of where and how to use mathematics and it must develop mathematical skills.

The 'second wave' is described by Skovsmose as that represented by the notion of 'ethnomathematics'.

His 'third wave' is that outline above and elaborated here:

"If a society is based on the use of manual tools, an idealistic interpretation of democratic competence remains plausible; no specific technological knowledge seems needed to evaluate the acts and decisions of the people in charge. Quite the contrary seems to be the case in a highly technological society. The content of democratic competence seems rapidly to acquire a tremendous complexity. In fact this competence seems to presuppose a certain amount of technological knowledge. How could anybody evaluate decisions which must take into consideration consequences of technological enterprises without a fair amount of technological knowledge? And in the information society..this technological knowledge is based in mathematics. "[41]

The *Digital Rhetorics* project, although focusing on literacy, listed four areas in which teachers must focus their understanding and pedagogy. Each of these simultaneously demand a numeracy focus:

- text-based including word processing and desk-top publishing,
- information-based including database and spreadsheet applications,
- programming-based, for example *LOGO* and
- games-based, for example *Super Mario Bros*.

To reiterate from Part 2 - Literacy, the literacies required - operational, cultural and critical - in the electronic age seem no less appropriate to numeracy in the context of technology.

Both related to and distinct from the numeracy issues arising from the electronic age, is the increasing awareness of the need for a visual literacy - or perhaps a visual numeracy?

Hannah (NZ) following Presmeg situates this need within a psychological framework, considering visualisation as a pedagogic tool and considers persons as having degrees of "mathematical visuality."[42]

Considering image as social practice Gunther Kress and Theo van Leeuwin's[43] influential grammatical descriptions of the ways that images mean is potentially useful for helping students to begin to 'read' images 'numerately'.

Following the priorities generated by much of the middle years movement in US and Australia, integration of the curriculum is supported by Barratt. The program at Norwood, a middle school in South Australia involves a "stable team of teachers" who work within subject areas, yet plan integrated units of work.

"We are involved in the International Baccalaureate Middle Years Program (IBMYP) a middle years' program from years 6 to 10. The program is based on curriculum integration and allows for subject expertise to co-exist with shared planning between teachers of different subject areas. Most of the teachers teach the one class for two subjects which allows for integration of content, and teachers of different subjects now share programs of work to discover connections and opportunities for integration."[44]

The Wollongong District Office Middle years consultant in NSW suggests:

"One successful organisational and curriculum change is the idea of teaming. Teaming in year 7 might involve teachers of Science and Mathematics in year 7 being responsible for teaching both subjects to their students."[45]

Literacy, Numeracy and Students with disabilities (2000) argues for additional attention to be given to the following, stemming from the *Report of the Numeracy Education Strategy Development Conference* (1997):

- development of awareness of the numeracy demands across key learning areas,
- understanding of the nature of numeracy,
- using identification and intervention strategies,
- addressing the numeracy needs of all students, and additional to these,
- that teachers need knowledge about new terms, new constructs and practices and technologies.[46]

[1] Kovacs, K. 1998 at p.1-2
[2] OECD 1997 at p.31
[3] OECD 2000 PISA Online
[4] Skovsmose, O. 1994 at p.63
[5] DETYA 2000 at p.14
[6] OECD 2000 IALS at p. (x)
[7] NSW DET 1998 Vol 3
[8] Curriculum Support for teaching in Mathematics 7-12 2000 Vol 5 No.2
[9] Spillane, J.P. 2000 at p.16
[10] Klein, M. 2000 at p.21
[11] ibid
[12] ibid at p.22
[13] Willis, S. 1998 at p.39
[14] Teese, R. et al 1998 at p.8
[15] ibid
[16] ibid at p.11
[17] Howard, P. 1998 at p. 7
[18] Milton, M. 2000 at p. 111
[19] van Kraayenoord, C. et al 2000
[20] Klein 2000 at p. 23
[21] Mulligan, J. 1999
[22] Anon EQ 1997 'Elizabeth and Me'
[23] Keast, S 1997 at p.176
[24] Howard, P. 1998 at p.4
[25] Klein, M. at p.27
[26] Barratt, R. 2000 at p.6
[27] Gould, P. 2000 at p.1-2
[28] ibid at p.24
[29] O'Neill, O. 1997 EQ at p.3
[30] Keast, S. 1997 at p. 175
[31] Morony, W. 1999
[32] Klein 2000 at p. 22
[33] Veel, R. 1999 at p.213
[34] Milton, M 2000 at p.122
[35] Goos, M. 1998 at p.232
[36] Shaugnessy, J.M.
[37] Lo Bianco & Freebody 1997 at p.92
[38] DETYA 2000 at p.15
[39] Landman, K 1997 at p.10-11
[40] Skovsmose, O. 1994 at p. 43
[41] ibid at p.57
[42] Hannah, K. 1998 at p. 233
[43] Kress & van Leeuwen
[44] O'Neill, O. 1997 at p.3
[45] Hadfield, C.
[46] van Kraayenoord, C. 2000 at p.7